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Численное моделирование и параллельные вычисления процессов тепломассопереноса при физико-химических воздействиях на неоднородный нефтяной пласт, вскрытый системой скважин

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В статье представлены математические и численные модели взаимосвязанных термо- и гидродинамических процессов эксплуатационного режима разработки единого нефтедобывающего комплекса при гидрогелевом заводнении неоднородного нефтяного пласта, вскрытого системой произвольно расположенных нагнетательных скважин и добывающих скважин, оснащенных погружными многоступенчатыми электроцентробежными насосами. Особенностью нашего подхода является моделирование работы специального наземного оборудования (станции управления погружными насосами и штуцерной камеры на устье добывающих скважин), предназначенного для регулирования режимов работы как всего комплекса в целом, так и его отдельных элементов.

Полная дифференциальная модель включает в себя уравнения, описывающие нестационарную двухфазную пятикомпонентную фильтрацию в пласте, квазистационарные процессы тепло- и массопереноса в трубах скважин и рабочих каналах погружных насосов. Специальные нелинейные граничные условия моделируют, соответственно, влияние диаметра дросселя на расход и давление на устье каждой добывающей скважины, а также частоты электрического тока на эксплуатационные характеристики погружного насосного узла. Разработка нефтяных месторождений также регулируется посредством изменения забойного давления каждой нагнетательной скважины, концентраций закачиваемых в нее гелеобразующих компонентов, их общих объемов и продолжительности закачки. Задача решается численно с использованием консервативных разностных схем, построенных на основе метода конечных разностей. Разработанные итерационные алгоритмы ориентированы на использование современных параллельных вычислительных технологий. Численная модель реализована в программном комплексе, который можно рассматривать как «интеллектуальную систему скважин» для виртуального управления разработкой нефтяных месторождений.

Ключевые слова: компьютерное моделирование, численные методы, параллельные алгоритмы, программные комплексы, многофазные потоки, добывающие и нагнетательные скважины, электроцентробежные насосы, неоднородный нефтяной пласт, гидрогелевое заводнение

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Numerical modeling and parallel computations of heat and mass transfer during physical and chemical actions on the non-uniform oil reservoir developing by system of wells

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The paper provides the mathematical and numerical models of the interrelated thermo- and hydrodynamic processes in the operational mode of development the unified oil-producing complex during the hydrogel flooding of the non-uniform oil reservoir exploited with a system of arbitrarily located injecting wells and producing wells equipped with submersible multistage electrical centrifugal pumps. A special feature of our approach is the modeling of the special ground-based equipment operation (control stations of submersible pumps, drossel devices on the head of producing wells), designed to regulate the operation modes of both the whole complex and its individual elements.

The complete differential model includes equations governing non-stationary two-phase five-component filtration in the reservoir, quasi-stationary heat and mass transfer in the wells and working channels of pumps. Special non-linear boundary conditions and dependencies simulate, respectively, the influence of the drossel diameter on the flow rate and pressure at the wellhead of each producing well and the frequency electric current on the performance characteristics of the submersible pump unit. Oil field development is also regulated by the change in bottom-hole pressure of each injection well, concentration of the gel-forming components pumping into the reservoir, their total volume and duration of injection. The problem is solved numerically using conservative difference schemes constructed on the base of the finite difference method, and developed iterative algorithms oriented on the parallel computing technologies. Numerical model is implemented in a software package which can be considered as the «Intellectual System of Wells» for the virtual control the oil field development.

Keywords: computer simulation, numerical methods, parallel algorithms, software packages, multi-phase multi-component flows, producing and injecting wells, electric centrifugal pumps, non-uniform oil reservoir, hydrogel flooding

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Introduction

The subject of the study is the interrelated thermo- and hydrodynamic processes in the unified oil-producing complex consisting of the oil reservoir, a system of arbitrarily located injecting wells and producing wells equipped with multistage electric submersible pumps (ESP).

Earlier a computer models were proposed to study such processes for cases of water-oil displacement [Konyukhov et al., 2017] and polymer flooding [Konyukhov et al., 2019] of the oil reservoir. In the second method, thickener in the form of an aqueous polymer solution of the desired concentration and volume is pumped into the oil reservoir within a certain time interval from all or from some injecting wells. Sorption of this thickener is irreversible and affects the permeability of the porous medium. An appearance of a sorbed polymer in well-developed areas with high permeability leads to the rise of the resistance factor proportionally to the polymer concentration, so that an effective permeability in these areas is significantly reduced. It results in the redirection of two-phase flows to the low-permeable zones of the reservoir and, as a consequence, more intensive and uniform displacement of oil by water in comparison with the usual flooding. In this regard, physical and chemical methods of improving the water-oil displacement are widely used in the extraction of oil from heterogeneous reservoirs.

In this article, we continue our research and generalize the mathematical, numerical and algorithmic models [Konyukhov et al., 2017; Konyukhov et al., 2019] for a more complicated physical and chemical method — hydrogel flooding. Such action on the reservoir provides the generation of the moving hydrogel fields directly in the areas of porous medium with different permeability and leads to a more significant improvement (compared with polymer flooding) in the uniformity of oil displacement and an increase in the main operating characteristics of oil production due to the redirection of fluid flows [Altunina et al., 2015; Chekalin et al., 2009]. The thickener R (hydrogel) is generated in the result of chemical reaction $\kappa_1 R_1 + \kappa_2 R_2 = \kappa R$ in mixture (water phase) between the aqueous solutions of two gelling components R_1, R_2 which injecting into the oil reservoir one after another with given time delay, where κ, κ_1, κ_2 are the constants of this reaction. The first component is sorbed more intensively and moves more slowly than the second one, so the moving hydrogel field begins to arise when the the second solution comes close to the first solution and begins to react chemically with him. Creation of the moving hydrogel field induces very complicated two-phase filtration flows in the non-uniform porous medium [Altunina et al., 2015; Imankulov et al., 2014; Chekalin et al., 2009; Chekalin et al., 2017; Walter et al., 1994].

In the operating mode of the oil fields development, the processes of heat and mass transfer in three-phase gas-water-oil mixtures moving in the wells and working channels of the electric centrifugal pumps are of quasi-stationary nature, but they are also very complex and accompanied by phase transitions during oil degassing in pipes and gas dissolution in pump channels, compressibility of phases, friction, gravity force, restructuring of gas-liquid flow, inversion of phases, drift motion of disperse components, heat exchange between flow and rock formation around the well, etc., see [Bratland, 2010; Hasan, Kabir, 1988]. Performance characteristics of submersible pump unit essentially depend on the properties of the pumped mixtures [Lyapkov, 1979]. In addition, at the present time the control for the current working modes of ESP are often realized with ground-based control stations (GCS) to analyze of the telemetering data (direct coupling) and to generate of the needed actions (back coupling) for improving of the work conditions of submersible equipment, up to its disable in emergency situations.

As a result of this, computation of these processes and optimization of the oil production should be done with taking account all above mentioned factors. These problems are very complex (see, e.g. [Barenblatt et al., 1984; Bear, 1988; Walter et al., 1994] and can be effectively solved on the basis of mathematical and numerical modeling using parallel computing technologies.

1. Mathematical model

Generalizing the unified mathematical models [Konyukhov et al., 2017; Konyukhov et al., 2019] for the case of hydrogel flooding results in the following conjugated system of non-linear differential equations:

a) two-dimensional equations of the two-phase five-component isothermal filtration in the porous reservoir $D_r = \{0 < x < L_x, 0 < y < L_y\}$:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \quad \vec{v} = \vec{v}_w + \vec{v}_o, \quad \vec{v}_w = -KH_r \frac{k_w^*(S)}{\mu_w(c)} \nabla P, \quad \vec{v}_o = -KH_r \frac{k_o^*(S)}{\mu_o} \nabla P; \tag{1}$$

$$mH_r \frac{\partial S}{\partial \tau} + \frac{\partial}{\partial x}(f v_x) + \frac{\partial}{\partial y}(f v_y) = 0; \tag{2}$$

$$mH_r \frac{\partial (c + a)}{\partial \tau} + \frac{\partial}{\partial x}(c f v_{w,x}) + \frac{\partial}{\partial y}(c f v_{w,y}) = \kappa \omega M, \quad a = \Gamma c; \tag{3}$$

$$mH_r \frac{\partial (c_1 + a_1)}{\partial \tau} + \frac{\partial}{\partial x}(c_1 f v_{w,x}) + \frac{\partial}{\partial y}(c_1 f v_{w,y}) = -\kappa_1 \omega M_1, \quad a_1 = \Gamma_1 c_1; \tag{4}$$

$$mH_r \frac{\partial (c_2 + a_2)}{\partial \tau} + \frac{\partial}{\partial x}(c_2 f v_{w,x}) + \frac{\partial}{\partial y}(c_2 f v_{w,y}) = -\kappa_2 \omega M_2, \quad a_2 = \Gamma_2 c_2; \tag{5}$$

$$c_1^{\nu_1} c_2^{\nu_2} = \chi c^{\nu}, \quad \mu_w(c) = \mu_w^0 (1 + \alpha_c c); \tag{6}$$

b) one-dimensional quasi-stationary momentum (force-balance) and energy equations for the disperse water-oil-gas flow with discrete phases (gas bubbles, water drops or oil drops) in the m-th producing well $D_m = \{0 < z \le H_m\}$, $m = \overline{1, M_p}$:

$$\frac{1}{f_c} \frac{d}{dz}(G_o \upsilon_o + G_g \upsilon_g + G_w \upsilon_w) = -\frac{dP}{dz} - \frac{2\tau_f}{r_c} + \rho g \cos\phi; \tag{7}$$

$$C_\beta^* \, \upsilon \frac{dT}{dz} = \alpha_\beta^* \, \upsilon T \frac{dP}{dz} - \frac{L}{f_c} \frac{G_o}{1 - C_s F} \frac{d(C_s F)}{dz} + Q_{\text{v}} + \frac{2}{r_c}(\tau_{cm} \upsilon - q_{cm}); \tag{8}$$

$$C_\beta^* = \rho_o \beta_o C_{p,o} + \rho_g \beta_g + C_{p,w} \rho_w \beta_w C_{p,w}, \quad \alpha_\beta^* = \alpha_{p,o} \beta_o + \alpha_{p,g} \beta_g + \alpha_{p,w} \beta_w;$$

c) one-dimensional heat and mass transfer equations for the three-phase flow in the channels of the m-th multistage electric centrifugal pumping unit $D_{e,m} = \{0 < \xi \le 1\}$, $m = \overline{1, M_p}$, located in the m-th well:

$$\frac{dP}{d\xi} = g \rho M_e H, \quad C_\varphi^* \frac{dT}{d\xi} = \left(\alpha_\varphi^* \, T + \frac{1 - \eta}{\eta} \right) \frac{dP}{d\xi} - \frac{L G_o}{1 - C_s F} \frac{d(C_s F)}{d\xi}, \tag{9}$$

$$C_\varphi^* = \rho_o \varphi_o C_{p,o} + \rho_g \varphi_g C_{p,g} + \rho_w \varphi_w C_{p,w}, \quad \alpha_\varphi^* = \alpha_{p,o} \varphi_o + \alpha_{p,g} \varphi_g + \alpha_{p,w} \varphi_w.$$

Here M_p is the number of the producing wells; τ is the time; the sub-indexes "w, o, g" denote characteristics of water, oil and gas, respectively; L_x and L_y are the length and the width of the reservoir; H_w is the depth of the m-th producing well; M_e is the number of the pump stages.

System (1)–(6) consists of the mass balance and momentum (the Darcy's law filtration) equations obtained with the use of the averaging procedure over the reservoir thickness without the regard to the capillary effects, gravity, compressibility of the phases and the porous medium. Mass transfer equations (3)–(6) are developed in [Chekalin et al., 2009; Chekalin et al., 2017] on the base of general principles of hydrogeochemistry [Walter et al., 1994]. The relationships (6) are written under the assumption that chemical reaction in a porous medium is very close to local equilibrium and the dynamic viscosity $\mu_w(c)$ of water solution depends only on the hydrogel concentration c.

In these equations, x and y are spatial coordinates; P is the pressure; S is the water saturation; c and c_i are the mass concentrations of components R and R_i in water solution ($i = 1, 2$); M and M_i are their molecular masses, $M = M_1 + M_2$; k_w^* and k_o^* are the functions of relative permeability of water and oil; v_x, v_y, $v_{w,x}$, $v_{w,y}$ are the projections of the filtration velocity vectors $\vec{v}$, $\vec{v}_w$ of the water-oil mixture and water phase to axis Ox and Oy; $K(x, y)$ and $m(x, y)$ are the absolute permeability and the dynamic porosity of the porous medium of the oil reservoir; $H_r(x, y)$ is its thickness; $f(S)$ is the fraction of water in the total two-phase flow (the Bacley–Leverett function); μ_o is the dynamic viscosity of oil; μ_w^0 is the viscosity of the hydrogel water solution at $c = 0$, α_c is an empirical coefficient; $a_i(c_i)$ and $a(c)$ are mass concentrations of the i-th gelling component and thickener in the absorbed (immovable) state. Their dependencies on concentration c and c_i are determined by the the Henry sorption isotherms, Γ, Γ_i are the Henry coefficients which can be obtained experimentally. Sorption is considered here as an effective total factor caused by the physical and chemical adsorption on the pore surface, dissolution in the formation rocks and mechanical retention in the narrowing of pore channels. The function ω is a difference between the numbers of acts at direct "+" and reverse "−" chemical reactions per unit time. Under the local equilibrium hypothesis, ω is taken as an unknown function in the equations (3)–(6) and must be computed from one of them [Walter et al., 1994].

The following group of equations (7), (8) is obtained in a framework of the Zuber–Findlay model and a quasi-stationary approximation (see, e.g. [Bratland, 2010; Hasan, Kabir, 1988]) for dispersed three-phase flow in the producing well when discrete components are the gas bubbles or drops of water (or oil) inside the continuous oil phase (or water phase, respectively). In these equations Oz is the vertical coordinate axis directed upwards the well from its beginning on the reservoir roof; P and T are the pressure and the temperature, identical for all phases; ρ is the mean multiphase mixture density; υ is the overall mixture flow velocity; ρ_i, υ_i, φ_i, β_i and G_i are the density, the actual velocity, the volumetric concentration, the consumption content and the mass flow rate (debit) of the i-th phase ($i = o, g, w$), averaged over the well pipe cross-section f_c of radius r_c; G is the overall mixture debit; $F(P, T)$ is the relative gas factor which is defined as the ratio of the mass of gas released from the oil phase at certain (P, T)-conditions to the total amount of the initially dissolved gas; C_s is the corresponding mass concentration of gas in the oil phase at $P > P_s$, where P_s is the saturation pressure; L is latent heat of gas dissolution into the oil phase; τ_{cm} and q_{cm} are the hydraulic friction and the heat flux density at the internal surface of the producing well; Q_{v} is the intensity of the external heat source distributed along the producing well; α_{pi} and C_{pi} are the coefficients of volumetric thermal expansion and volumetric elasticity of i-th phase; g is the gravity acceleration; $\phi(z)$ is the angle between the well profile and the axis Oz.

Equations (9) govern the thermal and hydrodynamic processes in the channels of the multi-stage pump. They were developed under the assumption that all the phases become highly dispersed in the pump stage and move without slippage in the result of the enormous rotation speed of blades, i.e. $\varphi_i = \beta_i$, $\upsilon_i = \upsilon$. In the equations (9) ξ is a share of the pump stages; H, $\eta = g\rho HQ/N$ and N are the head, the efficiency factor and the power consumption of the pump stage. These characteristics depend on the volumetric flow rate $Q = G/\rho$ and the effective viscosity μ of the three-phase mixture [Lyapkov, 1979] that can be significantly decrease by compressing the phases and dissolving the gas in the oil as the flow moves along the pump.

REMARK 1. In this paper we present only some relationships closing the equations (1)–(9). A whole set of special constitutive relations is too large and can be found in our publications (see e.g. [Chekalin et al., 2009; Chekalin et al., 2017; Konyukhov et al., 2017; Konyukhov et al., 2019]). Formulation of the boundary, initial and conjugation conditions for the system of differential equations is also discussed in details in these works.

The most important for modeling are formulated bellow a special non-linear boundary condition at the wellhead of the m-th producing well and relationships simulating the control actions on the operation modes of the m-th submersible pump and its electric motor by varying the frequency ω of the electric current with the use of the ground-based equipment [Konyukhov et al., 2019]:

$$P|_{z=H_w} = P_{lin} + 0.5 \cdot \varsigma_{dr}(d_{dr})\,\rho\upsilon^2\Big|_{z=H_w}; \tag{10}$$

$$Q_{\mathrm{w}} = Q^*_{\mathrm{w}}\omega/\omega^*, \quad H_{\mathrm{w}} = H^*_{\mathrm{w}}(\omega/\omega^*)^2, \quad N_m = N^*_m\omega/\omega^*, \quad N_{\mathrm{w}} = N^*_{\mathrm{w}}(\omega/\omega^*)^3. \tag{11}$$

Here the local resistance coefficient ς_{dr} of the regulating drossel is the function of its variable diameter d_{dr} affected on the values $P|_{z=H_w}$ and $\upsilon|_{z=H_w}$; P_{lin} is the constant line pressure behind the drossel; Q^*_{w}, H^*_{w}, and N^*_{w} are the volumetric flow rate, head and useful capacity of a certain pump stage during its operation on water at the nominal conditions at ω^*=50 Hz; N^*_m is the nominal consumed power of the motor at $\omega = \omega^*$; Q_{w}, H_{w}, N_{w}, and N_m are the similar characteristics of the stage and motor at $\omega \neq \omega^*$.

Oil field development is also controlled by the change in bottom-hole pressure of each injection well, concentration of the gel-forming components pumped into the reservoir, their volume and duration of injection.

2. Numerical model

The problem (1)–(11) is solved numerically using conservative finite difference schemes. In detail the numerical models for case of the water-oil displacement and polymer flooding are discussed in [Konyukhov et al., 2017; Konyukhov et al., 2019]. The following is a brief summary of some principal results of these works and their extension to the case of hydrogel flooding.

The pressure P is computed from the elliptic equation $div[\lambda(S,c)gradP] = 0$ that is obtained from equations (1) and approximated by the five-point finite difference scheme of the second order, $\lambda(S,c) = K(x,z)\cdot K^*(S,c)$. The corresponding system of algebraic equations with respect to P is solved by a special iterative method of high convergence [Konyukhov et al., 2017].

The mass balance equation (2) for the water saturation S is approximated in the all points (x_i, y_k) of the discrete domains D^h_r except the centers of wells with the use of points $(x_{i+0.5}, y_{k+0.5})$ of additional grid $\bar{D}^h_r$ by the upwind finite-difference scheme of the second order with respect to average integral values J_{ik} of this function:

$$mH_rh(J^{t+h_\tau}_{i,k} - J_{i,k})/h_\tau = V_{i+1/2,k} - V_{i-1/2,k} + V_{i,k+1/2} - V_{i,k-1/2}, \quad J_{i,k} = \frac{1}{h^2}\int\limits_{D_{i,k}} S\;dx\,dy. \tag{12}$$

Here h_τ is the time step; $h = h_x = h_y$ is the step of grid along the axes Ox and Oy in the discrete domain D^h_r; $V_{i\mp1/2,k} = h(fv_x)_{i\mp1/2,k}$; $V_{i,k\mp1/2} = h(fv_y)_{i,k\mp1/2}$; $D_{i,k}$ is the rectangular element of the grid D^h_r with the vertices located in the points $(x_{i\pm0.5}, y_{k\pm0.5}) \in \bar{D}^h_r$. The values $S_{i\pm1/2,k}$ and $S_{i,k\pm1/2}$ are defined by the linear-fractional interpolation through integral values J_{ik} in depending on the flow direction. Let, for example, $V_{i+1/2,k} < 0$, i.e. fluid flows through the border $\Gamma_{i+1/2,k}$ from the cell D_{ik} into the cell $D_{i+1,k}$. Then

$$S_{i+1/2,k} = \begin{cases} S^*, & S^* - \varepsilon^* \le J_{ik}, \\ F, & F \in [J_{i+1,k}, J_{ik}],\ S_* + \varepsilon_* \le J_{ik} < S^* - \varepsilon^*, \\ J_{ik}, & F \notin [J_{i+1,k}, J_{ik}],\ S_* + \varepsilon_* \le J_{ik} < S^* - \varepsilon^*, \\ S_*, & J_{ik} \le S_* + \varepsilon_*; \end{cases} \tag{13}$$

$$F = \begin{cases} 0.5\,(J_{i-1,k} + J_{ik})\,J_{ik}/J_{i-1,k}, & J_{i-1,k} \ge J_{ik}, \\ 0.5\left(1 + J_{ik} - (1 - J_{ik})^2/(1 - J_{i-1,k})\right), & J_{i-1,k} < J_{ik}, \end{cases}$$

where S_* and S^* are the values of irreducible and limiting water saturation; ε_* and ε^* are the semi-empirical parameters of the scheme (12), (13).

Before constructing the finite-difference schemes for equations (3)–(6), it is convenient to reduce them to the form of the transfer equation (2) by introducing the new functions:

$$C_j = \frac{c_j}{c_1^* M_1 \nu_j}, \quad j = 1,2; \quad C = \frac{c}{c_1^* M \nu}; \quad \gamma = \frac{\chi M^\nu (c_1^*)^{\nu-\nu_1-\nu_2}}{M_1^{\nu_1} M_2^{\nu_2}} (1+\Gamma_1)^{\nu_1} (1+\Gamma_2)^{\nu_2};$$

$$\Theta_j = S\left[(1+\Gamma_j)u_j + \left(\Gamma - \Gamma_j\right) C\right], \quad u_j = C + C_j, \quad j = 1,2. \tag{14}$$

Then, the equations (4)–(6) can be written in the following form:

$$mH_r \frac{\partial \Theta_j}{\partial t} + \frac{\partial}{\partial x}(u_j f v_{w,x}) + \frac{\partial}{\partial y}(u_j f v_{w,y}) = 0, \quad j = 1,2; \tag{15}$$

$$(\Theta_1/S - C(1+\Gamma))^{\nu_1} (\Theta_2/S - C(1+\Gamma))^{\nu_2} = C^\nu \gamma. \tag{16}$$

Now, with the use of the relationship (16) the function C can be expressed in terms of Θ_1 and Θ_2 as follows:

$$C = A - \sqrt{A^2 - \frac{\Theta_1 \Theta_2}{S^2(1+\Gamma)^2}}, \quad A = 0.5\left\{\frac{[\Theta_1 + \Theta_2]}{(1+\Gamma) S} + \gamma^{1/\nu} \frac{(1+\Gamma_1)(1+\Gamma_2)}{(1+\Gamma)^2}\right\}. \tag{17}$$

Further, by analogy with [Chekalin et al., 2009; Konyukhov et al., 2019], the equations (15) can be approximated by finite-difference schemes taking into account the possible arising both the leading and back fronts of concentrations, as well as changes in the magnitude jumps of these functions:

$$mH_r h \frac{U_{j,i,k}^{t+h_\tau} - U_{j,i,k}}{h_\tau} = W_{j,i+1/2,k} - W_{j,i-1/2,k} + W_{j,i,k+1/2} - W_{j,i,k-1/2}, \quad j = 1,2; \tag{18}$$

$$U_{j,i,k} = \frac{1}{h^2} \int\limits_{D_{i,k}} \Theta_j dx dy; \quad W_{j,i\mp1/2,k} = h(f u_j v_x)_{i\mp1/2,k}; \quad W_{j,i,k\mp1/2} = h(f u_j v_y)_{i,k\mp1/2}.$$

The grid functions $U_{j,i\pm1/2,k}$ and $U_{j,i,k\pm1/2}$ are defined on the base of parabolic interpolation through values U_{ik} also taking into account the flow direction. So, at $W_{j,i+1/2,k} < 0$,

$$U_{j,i+1/2,k} = \begin{cases} 0, & U_{j,i,k} \le \varepsilon_1, \\ 0, & \varepsilon_1 < U_{j,i,k} \le \varepsilon_2 U_{j,\max},\ C_{i+1,k} \le \varepsilon_1, \\ \Phi, & \begin{cases} \varepsilon_1 < U_{j,i,k} \le \varepsilon_2 U_{j,\max},\ C_{i+1,k} > \varepsilon_1, \\ \varepsilon_2 C_{\max} < U_{j,i,k} < \varepsilon_3 U_{j,\max}, \end{cases} \\ U_{j,\max}, & \varepsilon_3 U_{j,\max} \le U_{j,i,k}, \end{cases} \tag{19}$$

where $\Phi = 5/6 U_{j,i,k} + 1/3 U_{j,i+1,k} - 1/6 U_{j,i-1,k}$; $U_{j,\max} = \max\limits_{D_r^h} U_{j,i,k}$; ε_1, $1 - \varepsilon_3$ and ε_2 are the given parameters [Chekalin et al., 2009]. The first and second of them provide the calculation of $U_{j,i,k}$ within the assumed error. The value ε_2 is less than 2/3 of the jump concentration amplitude of the j-th component at its forward front.

The grid functions $U_{1,i,k}^{\tau+h_\tau}$ and $U_{2,i,k}^{\tau+h_\tau}$ are calculated from the equations (18), $i = 1,2$. After this the concentration $C_{i,k}^{\tau+h_\tau}$ can be determined from the relations

$$C_{i,k}^{\tau+h_\tau} = A - \sqrt{A^2 - U_{1,i,k}^{\tau+h_\tau} U_{2,i,k}^{\tau+h_\tau} / J_{i,k}^{\tau+h_\tau} / (1+\Gamma)^2}, \tag{20}$$

$$A = 0.5\left\{\left[U_{1,i,k}^{\tau+h_\tau} + U_{2,i,k}^{\tau+h_\tau}\right] / J_{i,k}^{\tau+h_\tau} / (1+\Gamma) + \gamma^{1/\nu} (1+\Gamma_1)(1+\Gamma_2)/(1+\Gamma)^2\right\}.$$

Needs to note that the water saturation S, concentrations C and C_i of hydrogel and the gel-forming components are multi-valued functions at the vertical boundaries of the producing and injecting wells. This specific feature of the problem solution can be taken into consideration by means of the special method [Chekalin et al., 2009], which allows us to determine the values of these functions in eight directions to the well (in the four directions along the coordinate axes and along the four diagonal directions). In this method the average integral values of S, C, and C_i are determined with the use of the special sector-shaped cells in the vicinity of the well borehole. For example, the finite-difference equations for the saturation S along axis Ox and along the diagonal direction of the first quadrant are written as follows:

$$\begin{aligned} J^{t+\tau}_{I+1,K} &= J_{I+1,K} + \frac{4h_\tau v_{x,I+3/2,K}}{3H_r mh}\left(f_{I+3/2,K} - f_{I+1,K}\right); \\ J^{t+\tau}_{I+1,K+1} &= J_{I+1,K+1} + +\frac{2h_\tau}{3H_r mh}[v_{x,I+\frac{3}{2},K+1}(f_{I+\frac{3}{2},K+1} - f_{I+1,K+1}) + v_{x,I+1,K+\frac{3}{2}}(f_{I+1,K+\frac{3}{2}} - f_{I+1,K+1})], \end{aligned} \tag{21}$$

where indexes $I = i_m$ and $K = k_m$ denote the grid coordinates (x_{i_m}, y_{k_m}) of the m-th producing well centre. The formulas for water saturation and concentrations along other directions can be written by analogy with equations (21). To provide the solution stability of the scheme (12), (18), (21) the time step h_τ is determined in according to the Courant–Friedrichs–Lewy criterion.

Systems of first order differential equations (7), (8), and (9) are solved with the use of the implicit Euler's schemes. However, their realization is significantly complicated because of a large number of the constitutive, usually non-linear relations, so that these equations can be solved only using the iterative procedures. Moreover, the number of the producing wells equipped with ESP-units on the oil field can reach several hundred, so the general number of grid points in all the wells may be much more than the number of grid points in the reservoir. This results in fact that the calculating times of filtration and heat mass transfer in the all wells become comparable.

Finally, in the presence of the drossel device on the head of the m-th producing well, the pressure $P|_{z=H_w}$ in the boundary condition (10) can significantly exceed the linear pressure P_{lin} at the input of the lead pipeline. This difference is caused by the second term of this relationship, which depends on both the drossel diameter and the three-phase flow characteristics. In this situation, the value P_{lin} is given, but the pressure, the mass flow rate of mixture, the volumetric concentration of water and gas at the bottom hole of the m-th producing wells are unknown, as well as the second term in (10). This leads to the fact that the unified system (1)–(11) must be solved by iterative method in a manner which assures to satisfy the boundary conditions (10) with given accuracy on the head of all producing wells. These features of the problem make it expedient and efficient to use parallel computing methods.

3. Software

The numerical model (12)–(21) is implemented in the program package "Oil-RWP". It allows us to simulate the interconnected processes in the unified complex "oil reservoir – system of wells" with simultaneous visualization of the results. Additionally, the special program "GCS" is developed to simulate an operation of the ground-based control stations and to regulate the work of the underground equipment in the producing wells. The Oil-RWP package sends telemetry data and values of the current operating parameters of the submersible motor to the GCS module (direct coupling). In turn, the station controller analyses incoming data and generates the required control parameters for electric submersible pump. These parameters are sent to the Oil-RWP package (back coupling).

4. Results of computations and conclusions

The results of computational experiments and their analysis were carried out for concrete oil-extracting complex, see [Konyukhov et al., 2017; Konyukhov et al., 2019]. It is shown that parallelization leads to the rise of the performance of calculations several times in comparison with sequential calculations in the following cases:

1) computation of two-phase five-component filtration in heterogeneous reservoir with the use of the grid consisting tens and hundreds thousand mesh points;

2) simultaneous computation of processes in every producing well equipped with its own electric pump at the grids with a large number of nodes;

3) visualization of the filtration process in the oil reservoir when the number of display pixels reaches several millions.

The practical results can be briefly formulated as follows:

1) the mathematical model and program package allows to realize the optimal control for the operating modes of each well, underground equipment, and, as a result, — the development of the oil reservoir;

2) the creation of moving hydrogel fields in the highly permeable areas of the heterogeneous reservoir significantly increases the uniformity of the oil displacement and improves its basic exploitation characteristics by redirecting filtration flows. The control for these moving fields is carried out in the package by the change in bottom-hole pressure of each injection well, concentration of the gel-forming components pumped into the reservoir, their volume and duration of injection.

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