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**MODELS OF ECONOMIC AND SOCIAL SYSTEMS**

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# A model for analyzing income inequality based on a finite functional sequence (adequacy and application problems)

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The paper considers the adequacy of the model developed earlier by the author for the analysis of income inequality and based on an empirically confirmed hypothesis that the relative (to the income of the richest group) income values of 20 % population groups in total income can be represented as a finite functional sequence, each member of which depends on one parameter — a specially defined indicator of inequality. It is shown that in addition to the existing methods of inequality analysis, the model makes it possible to estimate with the help of analytical expressions the income shares of 20 %, 10 % and smaller groups of the population for different levels of inequality, as well as to identify how they change with the growth of inequality, to estimate the level of inequality for known ratios between the incomes of different groups of the population, etc.

The paper provides a more detailed confirmation of the proposed model adequacy in comparison with the previously obtained results of statistical analysis of empirical data on the distribution of income between the 20 % and 10 % population groups. It is based on the analysis of certain ratios between the values of quintiles and deciles according to the proposed model. The verification of these ratios was carried out using a set of data for a large number of countries and the estimates obtained confirm the sufficiently high accuracy of the model.

Data are presented that confirm the possibility of using the model to analyze the dependence of income distribution by population groups on the level of inequality, as well as to estimate the inequality indicator for income ratios between different groups, including variants when the income of the richest 20 % is equal to the income of the poor 60 %, income of the middle class 40 % or income of the rest 80 % of the population, as well as when the income of the richest 10 % is equal to the income of the poor 40 %, 50 % or 60 %, to the income of various middle class groups, etc., as well as for cases, when the distribution of income obeys harmonic proportions and when the quintiles and deciles corresponding to the middle class reach a maximum. It is shown that the income shares of the richest middle class groups are relatively stable and have a maximum at certain levels of inequality.

The results obtained with the help of the model can be used to determine the standards for developing a policy of gradually increasing the level of progressive taxation in order to move to the level of inequality typical of countries with social oriented economy.

Keywords: inequality, income, model, distribution, inequality indicator, adequacy, sequence

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**МОДЕЛИ ЭКОНОМИЧЕСКИХ И СОЦИАЛЬНЫХ СИСТЕМ**

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# Модель для анализа неравенства доходов на основе конечной функциональной последовательности (проблемы адекватности и применения)

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Рассмотрены вопросы адекватности разработанной ранее автором модели для анализа неравенства доходов, основанной на эмпирически подтвержденной гипотезе о том, что относительные (по отношению к доходу наиболее богатой группы) величины дохода 20 % групп населения в совокупном доходе могут быть приближенно представлены в виде конечной функциональной последовательности, каждый член которой зависит от одного параметра — специально определенного показателя неравенства. Показано, что в дополнение к существующим методам анализа неравенства с помощью этой модели можно определить зависимость доли дохода 20 %, 10 % и более мелких групп населения от уровня неравенства, выявить особенности их изменения при росте неравенства, рассчитать уровень неравенства при известных соотношениях между доходами различных групп населения и др.

В работе приводится более подробное подтверждение адекватности предложенной модели по сравнению с полученными ранее результатами статистического анализа эмпирических данных о распределении доходов между 20%- и 10%-ми группами населения. Оно основано на анализе определенных соотношений между величинами квинтилей и децилей согласно предлагаемой модели. Проверка этих соотношений проведена по совокупности данных для большого числа стран. Полученные оценки подтверждают достаточно высокую точность модели.

Приведены данные, которые подтверждают возможность применения модели для анализа зависимости распределения доходов по группам населения от уровня неравенства, а также для оценки показателя неравенства для вариантов соотношений доходов между различными группами, в том числе когда доход 20 % наиболее богатых равен доходу 60 % бедных, доходу 40 % среднего класса или доходу 80 % остального населения, а также когда доход 10 % самых богатых равен доходу 40 %, 50 % или 60 % бедных, доходу различных групп среднего класса и др., а также для случаев, когда распределение доходов подчиняется гармоническим пропорциям и когда квинтили и децили, соответствующие среднему классу, достигают максимума. Показано, что доли дохода наиболее богатых групп среднего класса относительно стабильны и имеют максимум при определенных уровнях неравенства.

Полученные с помощью модели результаты могут быть использованы для определения нормативов при разработке политики поэтапного повышении уровня прогрессивного налогообложения с целью перехода к уровню неравенства, характерному для стран с социально ориентированной экономикой.

Ключевые слова: неравенство, доход, модель, распределение, показатель неравенства, адекватность, последовательность

# 1. Introduction

Much attention is now being paid to the problem of significant increases in income inequality, both globally and within countries, developed and developing (see [Piketty, Saez, 2006; Alvaredo et al., 2013, 2017; Atkinson et al., 2011; Milanovic, 2016; Piketty, 2014, 2020; Saez, 2015; Stiglitz, 2012; Mareeva, Tikhonova, 2016; Model …, 2018]). On average, as noted in [Chancel et al., 2022], the gap between the average income of the richest 10 % and the poorest 50 % within countries has almost doubled over the past 20 years. Excessive inequality leads to a decrease in labor productivity, deterioration in population health, increase in mortality, slowdown in economic growth, and instability in society that determines the high priority of research in this direction.

There are a number of methods for analysis of the income distribution. In addition to the Lorentz curve, the Gini index, the Pareto distribution function, lognormal distribution, Atkinson [Atkinson, 1970] and Theil [Theil, 1967] indices, and the Sen poverty index [Sen, 1976] are widely used. The ratio of income of the richest 10 % and the poorest 40 % or Palma index [Palma, 2011, 2014] as well as the ratio of income of the richest 1 % and the poorest 50 % , etc. are also often used [Chancel et al., 2022; Alvaredo, et al., 2013, 2017; Piketty, 2014, 2020; Saez, 2015].

These methods provide, as a rule, an aggregate estimate of inequality and poverty. However, it is interesting to determine how the share of income of each 20 % (quintiles) or 10 % (deciles) population group depends on the inequality level; to assess the inequality level at certain income ratios of different groups; to explain a reasonable range of inequality levels beyond which social tension in society increases significantly, etc.

The distribution of income of the population is usually given for 20 % and 10 % population groups (quintiles and deciles) and the level of inequality is characterized by the Gini index.

Using results of the analysis of a significant volume of statistical data on the income distribution in various countries over a long period of time, the author has come to the conclusion that there is a certain general law in the distribution of income by 20 % and 10 % population groups. This law is characterized by the following: the income ratios of the middle quintiles — the ratio of the fourth to the third, as well as the ratio of the third to the second are very close to each other. They are equal to the same value — $a$, which depends on the inequality level in each individual country. And the income ratios of the fifth (the richest) to the fourth, as well as the second to the first (the poorest) quintiles are very close to each other, and they are equal to the square of the middle quintiles ratio — $a^2$.

Similarly, as the analysis of statistical data for the distribution of income by 10 % population groups has shown, the income ratios of the average deciles are close to each other — the ratio of the eighth to the seventh deciles is equal to the ratio of the seventh to the sixth, etc. and to the ratio of the fourth to the third which is equal to $a^{1/2}$, and the income ratios of the tenth (the richest) to the ninth, as well as the second to the first (the poorest) deciles are also very close to each other, and they are also equal approximately to $a^2$, as the ratio of the extreme quintiles.

The results of a large volume of statistical data analysis show that this type of income distribution for equal population groups is the same for all countries, and the parameter $a$ depends on the inequality level in each country and increases with the growth of inequality, i. e. it can be used as an inequality indicator.

As a result of these observations, a model of income distribution based on a finite functional sequence was developed, the parameter of which is approximately equal to the ratio of incomes of two neighboring middle quintiles, depending on the level of inequality [Varshavsky, 2007, 2009, 2010, 2020, 2021].

It should be noted that the discovered law for the ratio of the extreme quintiles and deciles, i. e., the power law of distribution $N(a) = a^m$, $m \approx 2$, is also valid, for example, for ecosystems, as shown in [Camacho, Solé, 1999] where Zipf's law is used to describe the relationship between predator and prey organisms.

To confirm the validity of the discovered law, the author used several particular approaches in previous articles. However, even now some specialists ask for a better proof of the validity of the model. That is why this article provides a more detailed validation of the model based on the econometric analysis of the ratios determined by the model using the data from The World Income Inequality Database (WIID) for 2016 [The World].

In addition, new data are presented and the previously obtained data are systematized on the application of the proposed model and the possibility of obtaining additional results using this model.

## 2. A brief description of the model

As mentioned above, the following relationships were found for income ratios of quintiles as a result of the statistical data analysis:

$$\frac{Q4(a)}{Q3(a)} \approx \frac{Q3(a)}{Q2(a)} = a \text{ and } \frac{Q5(a)}{Q4(a)} \approx \frac{Q2(a)}{Q1(a)} = a^2.$$

Taking it into account, a model of income distribution was proposed for 20 % groups (quintiles). As was shown in the previous articles of the author, the relative (relative to the income of the richest group with number 5) income values of all five equal population groups in the total income can be approximated by a finite functional sequence in which the second and penultimate terms are removed:

$$A(a,5) = \{a^{-6}, a^{-4}, a^{-3}, a^{-2}, 1\},\ a \geq 1. \tag{1}$$

Each $i$-th term of this sequence is defined as the ratio of the income of the corresponding $i$-th population group to the income of the richest group ($i$ = 5) and depends on the parameter $a$ characterizing the level of inequality and defined as an inequality indicator.

The shares of the corresponding 20 % income groups $Qi(a)$, $i$ = 1, 2, ..., 5, in the total income are defined as

$$Q5(a) = \frac{1}{A(a^{-1})},\ Q4(a) = \frac{a^{-2}}{A(a^{-1})},\ Q3(a) = \frac{a^{-3}}{A(a^{-1})},\ Q2(a) = \frac{a^{-4}}{A(a^{-1})},\ Q1(a) = \frac{a^{-6}}{A(a^{-1})}, \tag{2}$$

where

$$A(a^{-1}) = 1 + a^{-2} + a^{-3} + a^{-4} + a^{-6} \tag{3}$$

is a characteristic polynomial of the finite power sequence $A(a,5)$; its features are described, for example, in [ Varshavsky, 2020, 2021].

The model (1)–(3) for 20 % groups is used as a basis for transition to models for 10 %, 5%, 2,5 %, 1,25 %, etc., population groups. For example, for 10 % population groups the deciles $Di(a)$, $i$ = 1, 2, ..., 10, are determined as:

$$D10(a) = \frac{\frac{a^m}{1+a^m}}{A(a^{-1})},\ D9(a) = \frac{\frac{1}{1+a^m}}{A(a^{-1})},\ D8(a) = \frac{\frac{b^{-3}}{1+b}}{A(a^{-1})},\ \ldots,$$

$$D3(a) = \frac{\frac{b^{-8}}{1+b}}{A(a^{-1})},\ D2(a) = \frac{b^{-12}\frac{a^m}{1+a^m}}{A(a^{-1})},\ D1(a) = \frac{b^{-12}\frac{1}{1+a^m}}{A(a^{-1})}, \tag{4}$$

where $b = a^{1/2}$, $A(a^{-1})$ is determined in accordance with (3), $m \simeq 1{,}8 - 2$ (for deriving analytical expressions we assume that $m = 2$; as the calculations show, for $D10(a)$ the difference in value at $m$ = 1,8 and 2 does not exceed 4,5 %, and for $D9(a)$ it is less than 10 % when $a$ < 1,55 and it does not exceed 18 % for $a$ = 1,6–2).

Similarly to (4), it is possible to construct a model for the distribution of income by 5 %, 2,5 % groups, etc., assuming that with each subsequent division of the richest and poorest group into two groups, the ratio of income of two pairs of extreme groups (left and right) remains approximately the same and is equal to $K = a^m$ [Varshavsky, 2020].

The inequality indicator $a$ can be determined by the least squares method using real data on income shares of 20 % groups for each individual country, as well as by estimating the ratio of real values of quintiles or deciles, as follows from (2) and (4):

$$a = \left(\frac{Q5}{Q1}\right)^{1/6}, \text{ or } a = \left(\frac{Q4}{Q2}\right)^{1/2}, \text{ or } a = \left(\frac{D10}{D1}\right)^{1/8} \text{ (for } m = 2). \quad (5)$$

The approximate relationship between the inequality indicator $a$ and the Gini index, as well as the corresponding quintile $\left(\frac{Q5(a)}{Q1(a)}\right)$ and decile share ratios $\left(\frac{D10(a)}{D1(a)}\right)$, are given in Table 1.

Table 1. Relation of inequality indicator $a$ with Gini index as well as quintile $\left(\frac{Q5(a)}{Q1(a)}\right)$ and decile $\left(\frac{D10(a)}{D1(a)}\right)$ share ratios [Varshavsky, 2020, 2021]

| a | 1,2 | 1,25 | 1,3 | 1,35 | 1,4 | 1,45 | 1,5 | 1,55 | 1,6 | 1,65 | 1,7 | 1,75 | 1,8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Gini*) | 22 | 26 | 30 | 34 | 37 | 40 | 44 | 47 | 49 | 52 | 55 | 57 | 59 |
| $\frac{Q5}{Q1}$ | 3,0 | 3,8 | 4,8 | 6,1 | 7,5 | 9,3 | 11,4 | 13,9 | 16,8 | 20,2 | 24,1 | 28,7 | 34,0 |
| $\frac{D10}{D1}$ | 4,3 | 6,0 | 8,2 | 11,0 | 14,8 | 19,5 | 25,6 | 33,3 | 42,9 | 54,9 | 69,8 | 88,0 | 110,2 |

*) Approximate estimate

## 3. Empirical confirmation of the model adequacy

This section presents the results of a detailed validation of the proposed model using WIID data for 2016, see the Appendix (it should be noted that the use of data from other sources, for example, the OECD Income Distribution Database [OECD], gave very similar results). It is based on the analysis of expressions (1)–(4), according to which the following relationships between the quintiles and deciles must be observed: $\frac{Q4}{Q3} = \frac{Q3}{Q2} = a$; $\frac{Q5}{Q4} = \frac{Q2}{Q1} = a^2$; $\frac{D10}{D9} = \frac{D2}{D1} = a^2$; $\frac{D5}{D4} = \frac{D4}{D3} = b = a^{1/2}$ and so on.

Thus, for each country, for the real values of quintiles, the following ratios of the form $\frac{Y}{X} = k_i$, where $k_i$ is determined in accordance with (2), that is $k_i$ = 1, 2, 3, 4, should be fulfilled:

$$\frac{\frac{Q4}{Q3}}{\frac{Q3}{Q2}} = \frac{\frac{Q5}{Q4}}{\frac{Q2}{Q1}} = \frac{\frac{Q5}{Q4}}{\frac{Q4}{Q2}} = 1, \text{ i. e. } k_i = 1;$$

$$-\frac{\ln\left(\frac{Q4}{Q5}\right)}{\ln(a)} = 2,\ k_i = 2;\ -\frac{\ln\left(\frac{Q3}{Q5}\right)}{\ln(a)} = 3,\ k_i = 3;\ -\frac{\ln\left(\frac{Q2}{Q5}\right)}{\ln(a)} = 4,\ k_i = 4;$$

$$-\frac{\ln\left(\frac{Q4}{Q5}\right)}{\ln\left(\frac{Q4}{Q3}\right)}=\frac{\ln\left(\frac{Q2}{Q1}\right)}{\ln\left(\frac{Q4}{Q3}\right)}=\frac{\ln\left(\frac{Q2}{Q1}\right)}{\ln\left(\frac{Q3}{Q2}\right)}=2,\ \ k_i\ =2;$$

$$\frac{\frac{Q4}{Q3}}{a}=\frac{\frac{Q3}{Q2}}{a}=1,\ \ k_i\ =1.$$

Similarly, for deciles the following equations must be fulfilled in accordance with (4):

$$\frac{\frac{D5}{D4}}{\frac{D4}{D3}}=\frac{\frac{D6}{D5}}{\frac{D5}{D4}}=\frac{\frac{D7}{D6}}{\frac{D6}{D5}}=\frac{\frac{D8}{D7}}{\frac{D7}{D6}}=\frac{\frac{D10}{D9}}{\frac{D2}{D1}}=1,\ \ k_i\ =1;$$

$$\frac{\ln\left(\frac{D10}{D9}\right)}{\ln(a)}=\frac{\ln\left(\frac{D2}{D1}\right)}{\ln(a)}=2,\ \ k_i\ =2,$$

where the inequality indicator is calculated using the real data as $a=\left(\frac{Q5}{Q1}\right)^{(1/6)}$.

In addition, the deviations of the ratios $\frac{D9}{D8}=\frac{a^2(1+a^{-1/2})}{1+a^2}=f_{98}(a)$ and $\frac{D3}{D2}=\frac{1+a^2}{1+a^{1/2}}=f_{32}(a)$ from their real values $\left(\frac{D9}{D8}\right)$ and $\left(\frac{D3}{D2}\right)$ determined in accordance with (3) and (4) at $m=2$ were also estimated using the data given in the Appendix.

The validation of these ratios for adequacy (coefficients $k_i$) based on real data (37 samples for 36 countries) was carried out using a regression equation $Y=kX+\varepsilon$, where the coefficient $k$ was determined using the ordinary least squares method (OLS). In Tables 2 and 3 the estimates of this coefficient ($k_{est}$) are given compared with the corresponding values of the coefficient $k_i$ determined on the basis of the model, the deviations are estimated as $\varepsilon=\frac{k_{est}-k_i}{k_i}\cdot 100\%$.

The results of the estimation that are given in Tables 2 and 3 provide convincing evidence of the adequacy of the model: the standard deviations and the discrepancy between the coefficients $k_{est}$ and $k_i$ are insignificant, and the coefficients of determination $R^2$ are close to 1.

The model adequacy was also confirmed using other methods.

Thus, in accordance with (1)–(4), the estimates of indicator $a$ for the income distribution by quintiles and deciles using the data for a large number of countries were carried out using the OLS method. The next regression equation was used for each country: $y=gx$, where $y_i=\ln\left(\frac{Qi}{Q5}\right)$, $i=1,...,5$ and $x_1=-6$, $x_2=-4$, $x_3=-3$, $x_4=-2$, $x_5=0$, and the inequality indicator $a$ was calculated as $a=e^g$.

Table 2. The results of estimation of the ratios between quintiles (coefficient $k_{est}$); data source: World Income Inequality Database (WIID), 2016, see the Appendix

| № | $Y$ | $X$ | $k_i$ | $k_{est}$ | Standard Error | Lower 95 % | Upper 95 % | $R^2$ | $\varepsilon$, % |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\frac{Q4}{Q3}$ | $\frac{Q3}{Q2}$ | 1 | 0,982 | 0,003 | 0,976 | 0,989 | 0,972 | –1,8 |
| 2 | $\frac{Q5}{Q4}$ | $\frac{Q2}{Q1}$ | 1 | 0,984 | 0,016 | 0,951 | 1,017 | 0,962 | –1,6 |
| 3 | $\frac{Q5}{Q4}$ | $\frac{Q4}{Q2}$ | 1 | 0,958 | 0,011 | 0,935 | 0,980 | 0,967 | –4,2 |
| 4 | $\ln\left(\frac{Q4}{Q5}\right)$ | $\ln(a)$ | –2 | –1,935 | 0,032 | –1,999 | –1,871 | 0,963 | –3,3 |
| 5 | $\ln\left(\frac{Q3}{Q5}\right)$ | $\ln(a)$ | –3 | –2,948 | 0,034 | –3,016 | –2,879 | 0,968 | –1,7 |
| 6 | $\ln\left(\frac{Q2}{Q5}\right)$ | $\ln(a)$ | –4 | –4,026 | 0,032 | –4,091 | –3,961 | 0,970 | 0,6 |
| 7 | $\ln\left(\frac{Q4}{Q5}\right)$ | $\ln\left(\frac{Q4}{Q3}\right)$ | –2 | –1,904 | 0,035 | –1,976 | –1,832 | 0,960 | –4,8 |
| 8 | $\ln\left(\frac{Q4}{Q5}\right)$ | $\ln\left(\frac{Q3}{Q2}\right)$ | –2 | –1,779 | 0,043 | –1,867 | –1,692 | 0,953 | –11,0 |
| 9 | $\ln\left(\frac{Q2}{Q1}\right)$ | $\ln\left(\frac{Q4}{Q3}\right)$ | 2 | 1,936 | 0,045 | 1,844 | 2,028 | 0,953 | –3,2 |
| 10 | $\ln\left(\frac{Q2}{Q1}\right)$ | $\ln\left(\frac{Q3}{Q2}\right)$ | 2 | 1,820 | 0,037 | 1,745 | 1,896 | 0,957 | –9,0 |
| 11 | $\frac{Q4}{Q3}$ | $a$ | 1 | 1,003 | 0,003 | 0,998 | 1,009 | 0,972 | 0,3 |
| 12 | $\frac{Q3}{Q2}$ | $a$ | 1 | 1,021 | 0,004 | 1,014 | 1,028 | 0,972 | 2,1 |

As an example, Figure 1 shows the real data (from the Appendix) and the estimates of the relative income shares in logarithms (relative to the income share of the richest 20 % $Q5$) for each quintile of the population of Norway, Italy, the USA and Germany. As can be seen in Figure 1, the extreme segments corresponding to $Q1$ and $Q5$ have a greater slope compared to the middle part of the income distribution. As mentioned above, this is typical of all countries and also confirms the validity of the model.

The standard deviation of the estimates from the real data ranges from 0,8 % (for Germany) to 6,5 % (for Italy), the coefficients of determination $R^2$ are higher than 0,989, which indicates a sufficiently high accuracy of the approximation. The estimates of the values of $Qi(a)$ and $Di(a)$, which were similarly carried out earlier using WIID data for 39 countries and also for 18 countries, were very close to their real values for all countries as well, and $R^2$ were no less than 0,975 [Varshavsky, 2010, 2020].

Table 3. The results of the estimation of ratios between deciles (coefficient $k_{est}$); data source: World Income Inequality Database (WIID), 2016, see Appendix

| | $Y$ | $X$ | $k_i$ | $k_{est}$ | Standard Error | Lower 95 % | Upper 95 % | $R^2$ | $\varepsilon$, % |
|---|---|---|---|---|---|---|---|---|---|
| 1 | $\frac{D4}{D3}$ | $b$ | 1 | 1,026 | 0,003 | 1,020 | 1,032 | 0,972 | 2,6 |
| 2 | $\frac{D5}{D4}$ | $b$ | 1 | 1,008 | 0,002 | 1,003 | 1,013 | 0,972 | 0,8 |
| 3 | $\frac{D6}{D5}$ | $b$ | 1 | 1,001 | 0,002 | 0,997 | 1,005 | 0,972 | 0,1 |
| 4 | $\frac{D7}{D6}$ | $b$ | 1 | 0,997 | 0,002 | 0,993 | 1,000 | 0,972 | –0,3 |
| 5 | $\frac{D8}{D7}$ | $b$ | 1 | 1,012 | 0,002 | 1,008 | 1,016 | 0,972 | 1,2 |
| 6 | $\frac{D5}{D4}$ | $\frac{D4}{D3}$ | 1 | 0,982 | 0,003 | 0,976 | 0,988 | 0,972 | –1,8 |
| 7 | $\frac{D6}{D5}$ | $\frac{D5}{D4}$ | 1 | 0,993 | 0,003 | 0,987 | 0,999 | 0,972 | –0,7 |
| 8 | $\frac{D7}{D6}$ | $\frac{D6}{D5}$ | 1 | 0,995 | 0,003 | 0,990 | 1,001 | 0,972 | –0,5 |
| 9 | $\frac{D8}{D7}$ | $\frac{D7}{D6}$ | 1 | 1,015 | 0,002 | 1,011 | 1,019 | 0,972 | 1,5 |
| 10 | $\frac{D10}{D9}$ | $\frac{D2}{D1}$ | 1 | 0,915 | 0,024 | 0,866 | 0,964 | 0,948 | –8,5 |
| 11 | $\ln\left(\frac{D10}{D9}\right)$ | $\ln(a)$ | 2 | 1,762 | 0,044 | 1,674 | 1,851 | 0,951 | –11,9 |
| 12 | $\ln\left(\frac{D2}{D1}\right)$ | $\ln(a)$ | 2 | 2,019 | 0,064 | 1,888 | 2,150 | 0,937 | 1,0 |
| 13 | $\frac{D9}{D8}$ | $f_{98}(a)$ | 1 | 1,017 | 0,003 | 1,011 | 1,023 | 0,972 | 1,7 |
| 14 | $\frac{D3}{D2}$ | $f_{32}(a)$ | 1 | 0,979 | 0,004 | 0,971 | 0,987 | 0,972 | –2,1 |
| 15 | $a_D$ | $a$ | 1 | 0,998 | 0,001 | 0,995 | 1,000 | 0,972 | –0,2 |

Note: in Tables 2 and 3 the inequality indicator was calculated using the real data for the first and fifth quintiles for each country as $a = \left(\frac{Q5}{Q1}\right)^{1/6}$; $b = a^{1/2}$; $a_D = \left(\frac{D10}{D1}\right)^{1/8}$; $\varepsilon = \frac{k_{est} - k_i}{k_i} \cdot 100\%$.

# 4. The possibility of obtaining additional results using the model

The proposed model expands the possibilities of income inequality research, which can be explained by the following.

First, using the model it becomes possible to determine the dependence of the different population groups income shares (quintiles, deciles, etc.) on the level of inequality, characterized by the inequality indicator $a$ or the Gini index. Tables 4 and 5 present the values of $Q1(a)$, ..., $Q5(a)$ and $D1(a)$, ..., $D10(a)$ in % calculated using (1)–(4) at different levels of inequality (for $a$ = 1,0–1,8).

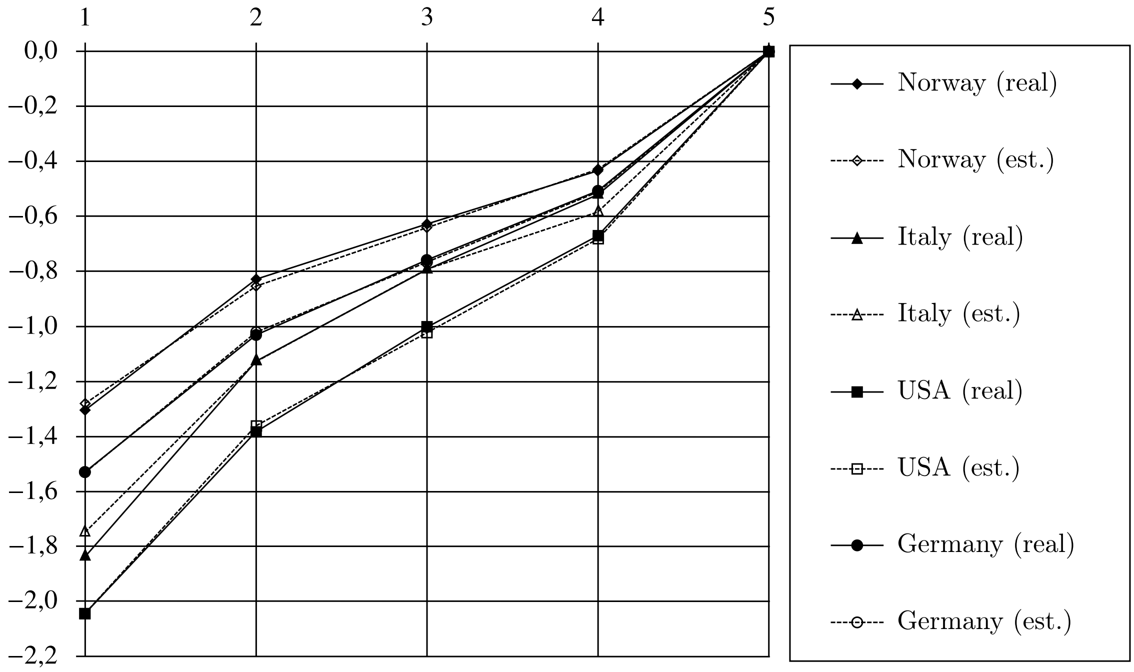


Fig. 1. The shares of income of each quintile of the population of Norway, Italy, the USA and Germany in relation to the share of income of the richest 20 % in logarithms: $\ln\left(\frac{Q1}{Q5}\right)$, $\ln\left(\frac{Q2}{Q5}\right)$, $\ln\left(\frac{Q3}{Q5}\right)$, $\ln\left(\frac{Q4}{Q5}\right)$, and $\ln\left(\frac{Q5}{Q5}\right)=0$ , real data and results of estimation (est.)

Table 4. Shares of quintiles $Q5(a)-Q1(a)$ in the total income at different levels of inequality, % (calculated using (1)–(3))

| Inequality indicator *a* | Gini[*)] | *Q*5 | *Q*4 | *Q*3 | *Q*2 | *Q*1 |
|---|---|---|---|---|---|---|
| 1 | 0 | 20,0 | 20,0 | 20,0 | 20,0 | 20,0 |
| 1,2 | 22 | 32,4 | 22,5 | 18,7 | 15,6 | 10,8 |
| 1,25 | 26 | 35,4 | 22,7 | 18,1 | 14,5 | 9,3 |
| 1,272 | 27 | 36,7 | 22,7 | 17,9 | 14,0 | 8,7 |
| 1,3 | 30 | 38,4 | 22,7 | 17,5 | 13,4 | 8,0 |
| 1,35 | 34 | 41,3 | 22,7 | 16,8 | 12,4 | 6,8 |
| 1,4 | 37 | 44,1 | 22,5 | 16,1 | 11,5 | 5,9 |
| 1,45 | 40 | 46,8 | 22,3 | 15,3 | 10,6 | 5,0 |
| 1,5 | 44 | 49,4 | 21,9 | 14,6 | 9,7 | 4,3 |
| 1,55 | 47 | 51,8 | 21,6 | 13,9 | 9,0 | 3,7 |
| 1,6 | 49 | 54,1 | 21,1 | 13,2 | 8,3 | 3,2 |
| 1,65 | 52 | 56,4 | 20,7 | 12,5 | 7,6 | 2,8 |
| 1,7 | 55 | 58,5 | 20,2 | 11,9 | 7,0 | 2,4 |
| 1,75 | 57 | 60,4 | 19,7 | 11,3 | 6,4 | 2,1 |
| 1,8 | 59 | 62,3 | 19,2 | 10,7 | 5,9 | 1,8 |

[*)] Evaluation, see Table 1.

Table 5. Shares of deciles $D10(a) - D1(a)$ in the total income at different levels of inequality, % (calculated using (1)–(4))

| Inequality indicator $a$ | Gini*) | $D10$ | $D9$ | $D8$ | $D7$ | $D6$ | $D5$ | $D4$ | $D3$ | $D2$ | $D1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |
| 1,2 | 22 | 19,1 | 13,3 | 11,7 | 10,7 | 9,8 | 8,9 | 8,2 | 7,4 | 6,4 | 4,4 |
| 1,25 | 26 | 21,6 | 13,8 | 12,0 | 10,7 | 9,6 | 8,6 | 7,7 | 6,8 | 5,7 | 3,6 |
| 1,272 | 27 | 22,7 | 14,0 | 12,0 | 10,7 | 9,5 | 8,4 | 7,4 | 6,6 | 5,4 | 3,3 |
| 1,3 | 30 | 24,1 | 14,3 | 12,1 | 10,6 | 9,3 | 8,2 | 7,2 | 6,3 | 5,0 | 3,0 |
| 1,35 | 34 | 26,7 | 14,6 | 12,2 | 10,5 | 9,0 | 7,8 | 6,7 | 5,8 | 4,4 | 2,4 |
| 1,4 | 37 | 29,2 | 14,9 | 12,2 | 10,3 | 8,7 | 7,4 | 6,2 | 5,3 | 3,9 | 2,0 |
| 1,45 | 40 | 31,7 | 15,1 | 12,2 | 10,1 | 8,4 | 7,0 | 5,8 | 4,8 | 3,4 | 1,6 |
| 1,5 | 44 | 34,2 | 15,2 | 12,1 | 9,9 | 8,1 | 6,6 | 5,4 | 4,4 | 3,0 | 1,3 |
| 1,55 | 47 | 36,6 | 15,2 | 12,0 | 9,6 | 7,7 | 6,2 | 5,0 | 4,0 | 2,6 | 1,1 |
| 1,6 | 49 | 38,9 | 15,2 | 11,8 | 9,3 | 7,4 | 5,8 | 4,6 | 3,6 | 2,3 | 0,9 |
| 1,65 | 52 | 41,2 | 15,1 | 11,6 | 9,1 | 7,1 | 5,5 | 4,3 | 3,3 | 2,0 | 0,8 |
| 1,7 | 55 | 43,4 | 15,0 | 11,4 | 8,8 | 6,7 | 5,2 | 4,0 | 3,0 | 1,8 | 0,6 |
| 1,75 | 57 | 45,6 | 14,9 | 11,2 | 8,5 | 6,4 | 4,9 | 3,7 | 2,8 | 1,6 | 0,5 |
| 1,8 | 59 | 47,6 | 14,7 | 11,0 | 8,2 | 6,1 | 4,6 | 3,4 | 2,5 | 1,4 | 0,4 |

*) Evaluation, see Table 1.

It is also possible to find a level of inequality for which certain proportions between some income groups or their combinations take place, for example, the ratio of the richest 10 % or 20 % to the poorest 60 %, 50 % or 40 %, etc. Thus, if the income of the richest 20 % is equal to the income of the poorest 60 %, that is, $Q5 = Q1 + Q2 + Q3,$ then the value of the inequality indicator $a$ is defined as the root of the equation $a^6 = a^3 + a^2 + 1,$ which is equal to $a$ = 1,304 (Gini ≈ 30). Similarly, it can be shown that the income of the richest 10 % is equal to the income of the poorest 50 % when the inequality indicator is equal approximately to $a \approx 1{,}35$ (Gini ≈ 34), etc.

Expressions for quintiles (2) and deciles (4) also show that the income shares of the middle-class groups ($Q4,\ D9,\ D8,\ D7$) have a maximum at certain levels of inequality. For example, the middle class 40 % $(D9 + Q4 + D6)$ = max at $a$ ≈ 1,33 (Gini ≈ 33), while $D9$ = max at $a$ ≈ 1,56 (Gini ≈ 47), $D8$ = max at $a$ ≈ 1,39 (Gini ≈ 37), $D7$ = max at $a$ ≈ 1,21 (Gini ≈ 22), $Q4$ = max at $a$ ≈ 1,30 (Gini ≈ 30).

Figure 2 shows how according to model (1)–(4) the share of income of the richest 10 %, the poorest 40 % and 50 %, as well as of the three middle-class groups 10 %, 20 % and 40 % depend on the level of the inequality indicator (for $m$ = 2). It should be noted that the income shares of middle-class groups are relatively stable, and this corresponds to the results of empirical data analysis in [Palma, 2014]. The share of the fourth quintile $Q4$ is particularly stable, but the richest 10 % of the middle class ($D9$) receive the greatest benefits from the inequality growth as its share increases to a rather high level up to $a$ ≈ 1,56 (Gini about 47), see Table 5.

Obviously, model (1)–(4) can be used for developing a policy of gradual inequality reduction to the level typical of socially oriented market economies. For example, as can be seen from Tables 4 and 5, reducing the inequality level from Gini = 35 to Gini = 25 will require an increase in the tax on income of the richest part of the population by 17–18 %, which is close to 16 %, which was determined in [Atkinson, 2015].

Secondly, using the model it is possible to prove the level of harmonious inequality when the inequality indicator is close to the level corresponding to the harmonious proportions: the Golden ratio (a special number found by dividing a line into two parts so that the longer part divided by the smaller part is also equal to the whole length divided by the longer part, this number is equal to $\phi$ ≈ 1,618) or

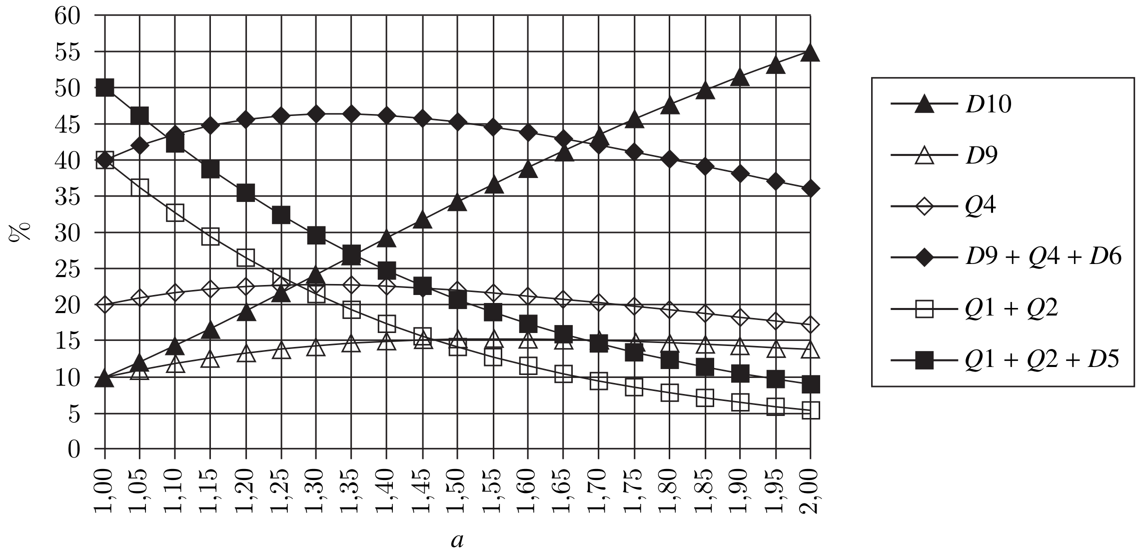


Fig. 2. The income shares of the richest 10 % ($D10$), the poorest 40 % ($Q1+Q2$) and 50 % ($Q1+Q2+D5$), as well as of three middle-class groups: 10 % ($D9$), 20 % ($Q4$) and 40 % ($D9+Q4+D6$) as function of the inequality indicator *a* according to the model of income distribution based on a finite functional sequence (1)–(4)

the Silver ratio (two quantities are in the Silver ratio if the ratio of the smaller of these two quantities to the larger quantity is the same as the ratio of the larger quantity to the sum of the smaller quantity and twice the larger quantity), which is characterized by the number $s = 1+2^{0,5} \approx 2{,}414$ [Corbalán, 2016; Gazalé, 1999]. The range of harmonious inequality, as shown in [Varshavsky, 2021], corresponds approximately to $a$ = 1,25–1,33 (Gini coefficient about 26–33) and is typical of the socially oriented market economies, where there is a significant redistribution of income through progressive taxation [Varshavsky, 2020, 2021].

As an example, we can consider the analytical expression for the Palma ratio [Palma, 2014], which can be easily found at $m = 2$ from (1)–(4):

$$P = \frac{D10}{Q1+Q2} = \frac{a^8}{(1+a^2)^2}, \tag{6}$$

where $a$ is the inequality indicator. The expression (6) shows that for $P = 1$ the value of the inequality indicator satisfies the condition of the golden ratio: $a^2 = \phi = 1{,}618$; $a = \phi^{1/2} = 1{,}272$ [Varshavsky, 2020, 2021]. If $P = 1$, i. e., if the share of income of the richest 10 % ($D10$) is equal to the share of income of the poorest 40 % ($Q1+Q2$), then simultaneously $D10 = Q4$ (this is seen in Fig. 2) and, as follows from (1)–(4), $\frac{Q5}{Q4} = \frac{Q2}{Q1} = \frac{D10}{D9} = \frac{D2}{D1} = a^2 = \phi = 1{,}618$, which corresponds to the golden ratio (variant 1). In this case we have the following distribution of income: $Q5$ = 36,7 %, $Q4$ = 22,7 %, $Q3$ = 17,9 %, $Q2$ = 14,0 %, $Q1$ = 8,7 %. Interestingly, in [Doyle, Stiglitz, 2014] it was proposed to reach this level of income inequality ($P = 1$) in the USA by 2030.

Approximately the same result is also observed when the income ratio of the richest 20 % and the remaining 80 % of population corresponds also to the golden ratio (variant 2), i. e., $Q5 = \phi^{-1}(100 - Q5)$, in this case $a \approx 1{,}3$.

Table 6 presents income distribution options characterized by different variants of the ratios between quintiles and deciles, as well as the corresponding inequality indicator $a$ and the Gini index, determined on the basis of the model (1)–(4) for $m = 2$. Among them are: the income of the rich 10 % is equal to the income of the lower middle class 20 %, or to the income of the poor 40 % (the golden

Table 6. Income distribution options, characterized by different quintile and decile ratios as well as corresponding values of the inequality indicator $a$ and Gini index, determined by the model (1)–(5)

| Income distribution options | Relationships between quintiles and deciles and their combinations | Indicator | |
|---|---|---|---|
| | | $a$ | Gini |
| The income of the richest 10 % is equal to the income of the lower middle class 20 % | $D10 = D7 + D6$ | 1,23 | 23 |
| Income distribution corresponds to the golden ratio (variant 1), $a^2 = \phi = 1{,}618$; $a = \phi^{1/2} = 1{,}272$. The income of the richest 10 % is equal to the income of the poorest 40 %, i. e., the Palma ratio is equal to 1: $P = \frac{D10}{Q1+Q2} = \frac{a^8}{(1+a^2)^2} = 1$ or $a^4 = 1 + a^2$, in this case $Q5 = 36{,}7$ %, $Q4 = 22{,}7$ %, $Q3 = 17{,}9$ %, $Q2 = 14{,}0$ %, $Q1 = 8{,}7$ % | $\frac{Q5}{Q4} = \frac{Q2}{Q1} = \frac{D10}{D9} = \frac{D2}{D1} = \phi = 1{,}618,$ $\frac{Q4}{Q3} = \frac{Q3}{Q2} = \phi^{1/2} = 1{,}272;$ $D10 = Q1 + Q2 = Q4;$ $D10 = \frac{1}{\phi}(D9 + D8 + D7)$ | 1,272 | 27 |
| The income ratio of the richest 20 % and the rest 80 % corresponds to the golden ratio (variant 2); $Q5 = 38{,}2$ %, $Q4 = 22{,}7$ %, $Q3 = 17{,}5$ %, $Q2 = 13{,}5$ %, $Q1 = 8{,}0$ % | $Q5 = \phi^{-1}(100 - Q5)$, $\phi^{-1} = 0{,}618$ | 1,30 | 30 |
| The income ratio of the richest 10 % and 30 % of the richest part of the middle class corresponds to the silver ratio $s = 1 + 2^{0,5} \approx \approx 2{,}414$ | $D10 = (s^{-0,5})(D9 + D8 + D7);$ $s^{-0,5} = 0{,}6436$ | 1,29 | 30 |
| The income of the richest 20 % and the rest 80 % corresponds to the silver ratio | $Q5 = (s^{-0,5})(Q4 + Q3 + Q2 + Q1);$ $s^{-0,5} = 0{,}6436$ | 1,31 | 31 |
| The income of the richest 20 % is equal to the income of the poorest 60 % | $Q5 = Q1 + Q2 + Q3$ | 1,30 | 30 |
| The income share of the middle class 40 % is equal to 40 % of the total income | $Q4 + Q3 = 40$ % | 1,31 | 31 |
| The income of the richest 20 % is equal to the income of the middle class 40 % | $Q5 = Q4 + Q3$ | 1,32 | 32 |
| The income of 30 % of the upper middle class is equal to the income of the poorest 60 % | $D9 + D8 + D7 = D6 + D5 + D4 + D3 + D2 + D1$ | 1,33 | 32 |
| The income of the richest 10 % is equal to the income of the poorest 50 % | $D10 = Q1 + Q2 + D5$ | 1,35 | 34 |
| The income of the richest 10 % is equal to the income of the lower middle class 30 % | $D10 = D7 + D6 + D5$ | 1,36 | 34 |
| The income of the richest 10 % is equal to the income of the upper middle class 20 % | $D10 = D9 + D8$ | 1,35 | 34 |
| The income of the richest 10 % is equal to the income of the middle and lower middle class 40 % | $D10 = Q3 + Q2$ | 1,38 | 36 |
| The income of the richest 10 % is equal to the income of the poorest 60 % | $D10 = Q1 + Q2 + Q3$ | 1,44 | 40 |
| The income of the richest 20 % is equal to the income of the remaining 80 % | $Q5 = 100 - Q5$ | 1,51 | 44 |

ratio, variant 1, the Palm index is equal to 1) or 50 %, to the income of the lower middle class 30 %, the upper middle class 20 %, the middle and lower middle class 40 %, the poor 60 %; variants in which the income ratio of the rich 10 % and of the richest part of the middle class 30 % corresponds to the silver ratio; the income ratio of the richest 20 % and the rest 80 % corresponds to the golden ratio (variant 2) or to the silver ratio; the income of the richest 20 % is equal to the income of the poor

60 %, to 40 % of the middle class or is equal to the income of the remaining 80 % of the population; the income share of 40 % of the middle class is equal to 40 % of the total income, the income of the upper middle class 30 % is equal to the income of the poor 60 %.

Finally, using the model, it is possible to determine an approximate range of inequality beyond which the social tension increases significantly and the level of stability in society decreases. This can be shown using the Shepley vector or by investigating transition processes. In addition, by modifying the Keynes model and using Caldor's ideas about the propensity to save, one can assess the influence of inequality on economic growth [Varshavsky, 2007, 2009, 2010].

## 5. Conclusion

Thus, the results of the analysis of the proposed model of income distribution based on a finite functional sequence, as well as the data given in the author's previous articles, confirm the adequacy of the model and the possibility of using it for analyzing the dependence of income distribution by population groups on the level of inequality.

With the help of the model, it becomes possible to determine the type of dependence of the income groups of the population on the inequality level, as well as to find analytical expressions for the ratios between different groups of quintiles and deciles, for example, for the widely used Palma ratio. The analysis of the model shows that the income shares of the richest middle class groups have a maximum at certain levels of inequality and the income shares of these groups are relatively stable.

The model allows us to confirm the level of harmonious inequality that is typical of socially oriented market economies, where there is a significant redistribution of income through progressive taxation. With its help, it is possible to find the level of inequality for certain income ratios between individual income groups or their combinations (for example, the income ratio of the richest 10 % of the population and of the poorest 50 % or 40 %, etc.).

The results obtained using the model can be used for developing a policy of gradually increasing the level of progressive taxation in order to reach the level of inequality typical of countries with socially oriented market economies.

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# Appendix

Table 7. Income distribution indicators and estimates of the inequality indicator *a*; data source: World Income Inequality Database (WIID), 2016

| | Country | Gini | $Q1$ | $Q2$ | $Q3$ | $Q4$ | $Q5$ | $D2$ | $D4$ | $D6$ | $D8$ | $D10$ | $a$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Austria | 27,2 | 8,8 | 14,2 | 18,2 | 22,8 | 36 | 5,5 | 7,6 | 9,6 | 12,1 | 22 | 1,26 |
| 2 | Belgium | 26,3 | 9,1 | 13,9 | 18,4 | 23,5 | 35,1 | 5,4 | 7,5 | 9,8 | 12,5 | 20,7 | 1,25 |
| 3 | Bulgaria | 38,3 | 5,6 | 11,4 | 16,2 | 22,6 | 44,2 | 3,9 | 6,3 | 8,8 | 12,2 | 28,9 | 1,41 |
| 4 | Canada | 30,6 | 7,4 | 13,2 | 17,9 | 23,4 | 38,1 | 4,7 | 7,2 | 9,6 | 12,5 | 23,1 | 1,31 |
| 5 | Croatia | 29,8 | 7,5 | 13,4 | 18,1 | 23,7 | 37,3 | 4,7 | 7,3 | 9,7 | 12,6 | 22,1 | 1,31 |
| 6 | Cyprus | 32,1 | 8,3 | 12,4 | 16,7 | 22,2 | 40,4 | 4,8 | 6,6 | 9 | 11,9 | 25,8 | 1,30 |
| 7 | Czechia | 25,1 | 10,1 | 14,6 | 17,9 | 22 | 35,4 | 6 | 7,6 | 9,4 | 11,7 | 21,6 | 1,23 |
| 8 | Denmark | 27,7 | 9 | 14,2 | 17,9 | 22,2 | 36,7 | 5,7 | 7,6 | 9,4 | 11,7 | 23,2 | 1,26 |
| 9 | Estonia | 32,7 | 7,1 | 12,1 | 17,1 | 23,9 | 39,7 | 4,5 | 6,6 | 9,3 | 13 | 23,7 | 1,33 |
| 10 | Finland | 25,4 | 9,9 | 14,3 | 18 | 22,4 | 35,4 | 5,7 | 7,6 | 9,5 | 11,9 | 21,5 | 1,24 |
| 11 | France | 29,3 | 8,9 | 13,7 | 17,2 | 21,6 | 38,5 | 5,3 | 7,3 | 9,1 | 11,5 | 24,7 | 1,28 |
| 12 | Germany | 29,5 | 8,2 | 13,5 | 17,7 | 22,8 | 37,8 | 5,1 | 7,3 | 9,4 | 12,2 | 23,3 | 1,29 |
| 13 | Greece | 34,3 | 6,2 | 12,5 | 17,3 | 23,5 | 40,5 | 4,2 | 6,8 | 9,2 | 12,7 | 25,1 | 1,37 |
| 14 | Hungary | 28,2 | 8,6 | 13,9 | 17,8 | 22,9 | 36,9 | 5,4 | 7,4 | 9,4 | 12,3 | 22,6 | 1,27 |
| 15 | Iceland | 24,1 | 10,4 | 14,9 | 18,1 | 22 | 34,6 | 6,1 | 7,8 | 9,5 | 11,6 | 21,2 | 1,22 |
| 16 | Ireland | 29,5 | 8,6 | 13,1 | 17,6 | 22,9 | 37,9 | 5,1 | 7 | 9,4 | 12,3 | 23,5 | 1,28 |
| 17 | Israel | 34,7 | 6,1 | 11,7 | 17,3 | 24,2 | 40,6 | 3,9 | 6,5 | 9,4 | 13,1 | 24,5 | 1,37 |
| 18 | Italy | 33,1 | 6,3 | 12,8 | 17,9 | 23,5 | 39,5 | 4,5 | 7 | 9,6 | 12,6 | 24,4 | 1,36 |
| 19 | Latvia | 34,5 | 6,6 | 12 | 16,9 | 23,3 | 41,2 | 4,3 | 6,5 | 9,1 | 12,6 | 25,6 | 1,36 |
| 20 | Lithuania | 37 | 6,1 | 11,4 | 16,2 | 22,9 | 43,3 | 4,1 | 6,3 | 8,7 | 12,5 | 27,7 | 1,39 |
| 21 | Luxembourg | 31 | 7,8 | 13,1 | 17,3 | 22,8 | 39,1 | 4,8 | 7 | 9,3 | 12,3 | 24,1 | 1,31 |
| 22 | Macedonia, | 33,6 | 6 | 12,5 | 17,7 | 24,2 | 39,6 | 4,2 | 6,9 | 9,7 | 13,1 | 23,7 | 1,37 |
| 23 | Malta | 28,5 | 8,9 | 13,3 | 17,7 | 22,8 | 37,3 | 5,2 | 7,2 | 9,4 | 12,2 | 22,8 | 1,27 |
| 24 | Netherlands | 26,9 | 9,2 | 14,1 | 17,9 | 22,6 | 36,1 | 5,6 | 7,5 | 9,4 | 12 | 22,1 | 1,26 |
| 25 | Norway | 25 | 9,4 | 15,1 | 18,5 | 22,4 | 34,6 | 5,9 | 8 | 9,7 | 11,8 | 21,0 | 1,24 |
| 26 | Poland[1)] | 28,8 | 8,5 | 13,8 | 17,6 | 22,5 | 37,6 | 5,3 | 7,4 | 9,4 | 12,0 | 23,3 | 1,28 |
| 27 | Poland[2)] | 29,8 | 7,9 | 13,4 | 17,7 | 23,1 | 37,8 | 5 | 7,2 | 9,4 | 12,4 | 22,9 | 1,30 |
| 28 | Portugal | 33,9 | 7 | 12,4 | 16,7 | 22,6 | 41,4 | 4,5 | 6,7 | 8,9 | 12,3 | 25,9 | 1,34 |
| 29 | Slovakia | 24,3 | 9,3 | 15,2 | 18,8 | 23 | 33,7 | 5,9 | 8,1 | 9,8 | 12 | 19,9 | 1,24 |
| 30 | Slovenia | 24,4 | 9,5 | 14,9 | 18,7 | 22,9 | 34 | 5,7 | 7,9 | 9,8 | 12,1 | 20,1 | 1,24 |
| 31 | Spain | 34,5 | 6,2 | 12,2 | 17,3 | 23,7 | 40,7 | 4,2 | 6,7 | 9,3 | 12,9 | 24,9 | 1,37 |
| 32 | Sweden | 27,6 | 8,5 | 14,1 | 18,4 | 23,2 | 35,9 | 5,4 | 7,6 | 9,7 | 12,3 | 21,8 | 1,27 |
| 33 | Switzerland | 29,4 | 8,6 | 13,5 | 17,4 | 22,4 | 38,1 | 5,2 | 7,2 | 9,3 | 11,9 | 23,9 | 1,28 |
| 34 | Taiwan (China) | 30,3 | 8,2 | 13,2 | 17,2 | 22,8 | 38,7 | 5,0 | 7,07 | 9,1 | 12,3 | 23,7 | 1,30 |
| 35 | United Kingdom | 31,5 | 7,7 | 13 | 17,2 | 22,9 | 39,2 | 5 | 7 | 9,2 | 12,3 | 24,3 | 1,31 |
| 36 | United States | 38,3 | 5,7 | 11,1 | 16,3 | 22,7 | 44,3 | 3,8 | 6,1 | 8,8 | 12,4 | 28,5 | 1,41 |
| 37 | Uruguay | 36 | 7,0 | 11,4 | 16 | 22,3 | 43,3 | 4,2 | 6,2 | 8,6 | 12,2 | 27,7 | 1,36 |

1) Luxembourg Income Study, LIS 2018.
2) Eurostat 2018.