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ANALYSIS AND MODELING OF COMPLEX LIVING SYSTEMS

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A study on the dynamics of pest population with biocontrol using predator, parasite in presence of awareness

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The coconut tree is often mentioned as the “tree of life” due to its immense benefits to the human community ranging from edible products to building materials. Rugose spiraling whitefly (RSW), a natural enemy seems to be a major threat to farmers in bringing up these coconut trees. A mathematical model to study the dynamics of pest population in the presence of predator and parasite is developed. The biologically feasible equilibrium points are derived. Local asymptotic stability as well as global asymptotic stability is analyzed at the points. Furthermore, in order to educate farmers on pest control, we have added the impact of awareness programs in the model. The conditions of existence and stability properties of all feasible steady states of this model are analyzed. The result reveals that predator and parasite play a major role in reducing the immature pest. It also shows that pest control activities through awareness programs further reduce the mature pest population which decreases the egg laying rate which in turn reduces the immature population.

Keywords: mathematical model, rugose spiralling whitefly, predator, parasite, stability

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1. Introduction

The coconut trees belong to the species of palm family that grows extensively in tropical climates. India is the world's largest producer of coconuts and related products, with the southern states of the nation making the largest contributions. Around 90 % of the nation's coconut plantations are contributed by the five major states. Pests that strongly affect the growth of the tree and its ability to produce coconut is one of the biggest problems. Since 2016, coconut trees in India have been affected by the exotic pest known as rugose spiralling whitefly [Elango, Nelson, Aravind, 2020]. In its native range, it primarily infests broad-leaved hosts like coconut trees. The pest shares some superficial similarities with the invasive spiralling whitefly Aleurodicus dispersus, which arrived in India in the middle of the 1990s, in terms of behavior and general appearance. It affects the host plant by sucking up the water and vital nutrients from the leaves. It also creates honeydew, which coats the lower leaves and encourages the development of sooty mold. Even though sooty mould is not a plant disease, its presence on the top of a leaf may hinder a plant's ability to synthesize oxygen [Alagar et al., 2020]. Field surveys were carried out in the southern districts of Tamil Nadu which were severly affected by these pests. There were nine species of predators, one species of nymphal parasitic aphelinid observed. The aphelinid parasitoid Encarsia guadeloupae is one of RSW's most important naturally occurring adversaries since it significantly reduces the species' population [Elango, Nelson, 2020; Suriya et al., 2021]. N. C. Rao et al. examined the feeding behavior of Pseudomallada astur in relation to the Rugose spiralling whitefly and its functional response. Each larval instar agreed with the Type III functional response. A possible predator of whiteflies infesting coconut trees has been identified as Pseudomallada astur. After inoculating the Pseudomallada astur eggs in the whitefly-infected leaves, a field evaluation revealed a noticeably decreased population of whiteflies [Rao, Ramani, Bhagavan, 2020; Chalapathi Rao et al., 2022].

Using a mathematical model, the dynamics of population interaction can be examined. The concept and idea of mathematical models have been discussed in relation to epidemiology [Allen et al., 2008]. An epidemiological model with vector population dynamics was utilized to study the African cassava mosaic virus disease [Holt et al., 1997]. A model for biocontrol of pests with a Holling type II functional response has been developed and its dynamical behavior has been studied in [Pathak, Maiti, 2012]. Stage structure of prey species is taken into account in a predator-prey interaction model that includes juvenile prey, adult prey, and predator with ratio-dependent functional response. Global dynamics has been analyzed with the geometrical approach [Panja, Jana, Mondal, 2021]. The effect of environmental toxins has been studied using the nonautonomous model [Mandal et al., 2020]. In [Suganya, Senthamarai, 2022b], an analysis has been carried out for the interaction between rugose spiraling whitefly and coconut trees. The findings suggested that reducing the contact rate using management methods can lessen both the likelihood of infecting healthy trees and severity of the disease. In [Basir, Banerjee, Ray, 2019], research has been conducted on the significance of agricultural awareness for pest control. The effects of farming awareness-based interventions on the dynamics of mosaic disease have been investigated [Basir, Blyuss, Ray, 2018; Basir et al., 2018]. The effect of public awareness on the elimination of environmental pollutants that harm the planktonic system has been modeled and analyzed [Mandal, Tiwari, Pal, 2022]. The effect of the media campaign on reducing carbon dioxide emissions from automobiles has been analyzed and the results reveal that the rate of awareness program implementation through the media campaign has a significant impact on the mitigation of carbon dioxide [Sundar, Mishra, Naresh, 2018]. According to the research in this literature, awareness efforts may eventually reduce or even stop the spread of the mosaic disease. It is also advised that if the advertisement awareness efforts are carried out in short time intervals, the fading of knowledge among farmers and the delay in implementation might be prevented. The stability theory of differential equations was used to do the study for the spread of infectious diseases [Misra, Sharma, Shukla, 2011]. The study in [Suganya, Senthamarai, 2022a] analyzes the dynamics of phytoplankton, zooplankton in the presence of the nanoparticles. A stability analysis has been carried out and analytical approximation is obtained in [Vijayalakshmi, Senthamarai, 2022].

In this study, we analyze the dynamics of the pest population using biocontrol agents like predator, parasite along with awareness. The parameter values are chosen based on experimental results and data given in the literature. Local and global stability has been analyzed at the equilibrium points. Finally, we numerically investigate the parameter effect on the system and compile the results.

2. Mathematical formulation

The dynamics of rugose spiraling whitefly affecting coconut trees has been analyzed in [Suganya, Senthamarai, 2022b]. Here, we develop a mathematical model for pest control using biocontrol agents. To study the interaction of pest with its predator and parasite, we consider immature rugose spiralling whitefly (pest) as $x_1(t)$ leaf^{-1}, mature pest as $x_2(t)$ leaf^{-1}, Pseudomallada astur (predator) as $P(t)$ leaf^{-1} and Encarsia guadeloupae (parasite) as $S(t)$ leaf^{-1} with time t. The model is given by

$$\begin{aligned}
\frac{dx_1}{dt} &= \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) - \frac{ax_1^2P}{c^2 + x_1^2} - \omega x_1 S - \gamma x_1,\\
\frac{dx_2}{dt} &= \gamma x_1 - \mu_1 x_2,\\
\frac{dP}{dt} &= \frac{abx_1^2P}{c^2 + x_1^2} - \mu_2 P + \beta,\\
\frac{dS}{dt} &= \omega\omega_1 x_1 S - \mu_3 S - \mu_4 S^2,
\end{aligned} \tag{1}$$

with the initial conditions

$$x_1(0) > 0, \quad x_2(0) > 0, \quad P(0) > 0, \quad S(0) > 0. \tag{2}$$

Let α denote the egg laying rate (fecundity rate) of mature pest, a — the predation rate, ω — the parasitization rate, and γ — the conversion of immature pest to mature pest. The mortality rate of pest is denoted by μ_1, saturation constant is c, and carrying capacity of pest per leaf is denoted by k. Let b denote the conversion rate of predator, μ_2 — the death rate of predator, and the number of predators are added to the system at a rate β. The conversion rate of parasite is denoted by ω_1, the death rate is μ_3 and the density-dependent death rate of parasite is denoted by μ_4.

In order to study the pest control via awareness programs, we have incorporated the awareness term in our model as well. The awareness program is denoted by $A(t)$ and the model is given by

$$\begin{aligned}
\frac{dx_1}{dt} &= \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) - \frac{ax_1^2P}{c^2 + x_1^2} - \omega x_1 S - \gamma x_1,\\
\frac{dx_2}{dt} &= \gamma x_1 - \mu_1 x_2 - \delta A x_2,\\
\frac{dP}{dt} &= \frac{abx_1^2P}{c^2 + x_1^2} - \mu_2 P + \beta,\\
\frac{dS}{dt} &= \omega\omega_1 x_1 S - \mu_3 S - \mu_4 S^2,\\
\frac{dA}{dt} &= r_0 + hx_2 - \eta A,
\end{aligned} \tag{3}$$

with the initial conditions

$$x_1(0) > 0, \quad x_2(0) > 0, \quad P(0) > 0, \quad S(0) > 0, \quad A(0) > 0, \tag{4}$$

where δ denotes the death rate of mature pest to due awareness activities. Let the awareness level increase at a rate r_0 through media activities. The rate of local awareness, h, is proportional to the number of mature pest and η denotes fading of awareness due to falling of importance.

3. Mathematical analysis of the system (1)

3.1. Positivity and boundedness

From system (1) it can be seen that

$$\left.\frac{dx_1}{dt}\right|_{x_1=0} = \alpha x_2\left(1 - \frac{x_2}{k}\right) \geqslant 0, \quad \left.\frac{dx_2}{dt}\right|_{x_2=0} = \gamma x_1 \geqslant 0, \quad \left.\frac{dS}{dt}\right|_{P=0} = \beta \geqslant 0, \quad \left.\frac{dP}{dt}\right|_{S=0} = 0.$$

The total pest population $N = x_1 + x_2$ satisfies

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) - \frac{a x_1^2 P}{c^2 + x_1^2} - \omega x_1 S - \mu_1 x_2,$$

$$\frac{dN}{dt} + \zeta x_1 + \zeta x_2 \leqslant \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) + \zeta x_1 + \zeta x_2,$$

$$\frac{dN}{dt} + \zeta N \leqslant -\frac{\alpha x_2^2}{k} + (\alpha + \zeta)x_2.$$

It is seen that $-\frac{\alpha x_2^2}{k} + (\alpha + \zeta)x_2$ is quadratic in x_2 and its maximum value is $\frac{(\alpha+\zeta)^2 k}{4\alpha}$

$$\frac{dN}{dt} + \zeta N \leqslant l,$$

where $l = \frac{(\alpha+\zeta)^2 k}{4\alpha}$.

$$0 \leqslant N(t) \leqslant e^{-\zeta t}\left(N(0) - \frac{l}{\zeta}\right) + \frac{l}{\zeta}.$$

As $t \to \infty$, $N(t) \to \frac{l}{\zeta}$ since $\sup\limits_{t\to\infty} N(t) = \frac{l}{\zeta}$.

Since x_1 is bounded,

$$\frac{dP}{dt} + \mu_2 P \leqslant mP + \beta.$$

Then $\sup\limits_{t\to\infty} P(t) = \frac{\beta}{\mu_2 - m}$

$$\frac{dS}{dt} + \mu_3 S \leqslant RS - \mu_4 S.$$

Thus, $\sup\limits_{t\to\infty} S(t) = \frac{\mu_4^2}{4(R-\mu_3)}$. Thus, the biologically feasible region of the system (1) is the following positive invariant set:

$$\Omega = \left\{(x_1, x_2, P, S) \in \mathbb{R}_+^4 \mid 0 \leqslant x_1, x_2 \leqslant \frac{l}{\zeta}, \; P \leqslant \frac{\beta}{\mu_2 - m}, \; S \leqslant \frac{\mu_4^2}{4(R - \mu_3)}\right\}.$$

3.2. Existence of equilibria

The system (1) possesses three nonnegative equilibrium points. They are:

- pest and parasite free equilibrium: $E_0 = \left(0,\, 0,\, \frac{\beta}{\mu_2},\, 0\right)$;
- parasite free equilibrium: $\widetilde{E} = (\widetilde{x}_1, \widetilde{x}_2, \widetilde{P}, 0)$ where

$$\widetilde{x}_2 = \frac{\gamma \widetilde{x}_1}{\mu_1}, \quad \widetilde{P} = \frac{\beta\left(c^2 + \widetilde{x}_1^2\right)}{\mu_2\left(c^2 + \widetilde{x}_1^2\right) - ab\widetilde{x}_1^2},$$

$\widetilde{x}_1$ is the positive root of the equation,

$$\widetilde{x}_1^3\left(\frac{\alpha}{k\mu_1}(1+\gamma)\right) - \widetilde{x}_1^2\left(\frac{\alpha}{\mu_1} - 1\right) + \widetilde{x}_1\left(\frac{\alpha c^2}{k\mu_1}(1+\gamma)\right) - \frac{\alpha c^2}{\mu_1} + c^2 = 0;$$

- endemic equilibrium: $E_* = \left(x_1^*,\, x_2^*,\, P^*,\, S^*\right)$ where

$$x_2^* = \frac{\gamma x_1^*}{\mu_1}, \quad P^* = \frac{\beta\left(c^2 + x_1^{*2}\right)}{\mu_2\left(c^2 + x_1^{*2}\right) - abx_1^{*2}}, \quad S^* = \frac{\omega\omega_1 x_1^* - \mu_3}{\mu_4},$$

x_1^* is the positive root of the equation,

$$x_1^{*3}\left(\frac{\alpha\gamma}{k\mu_1} + \frac{\alpha\gamma^2}{k\mu_1} + \frac{\omega^2\omega_1}{\mu_4}\right) + x_1^{*2}\left(\gamma - \frac{\alpha\gamma}{\mu_1} - \frac{\omega\mu_3}{\mu_4}\right) +$$
$$+ x_1^*\left(\frac{\alpha\gamma c^2}{k\mu_1} - \frac{\alpha\gamma}{\mu_1} + \frac{\alpha\gamma^2 c^2}{k\mu_1} + \frac{\alpha\gamma^2 c^2}{k\mu_1} + ax_1^* P^* + \frac{\omega^2\omega_1 c^2}{\mu_4}\right) - \frac{\alpha\gamma c^2}{\mu_1} - \frac{\omega\mu_3 c^2}{\mu_4} + \gamma c^2 = 0.$$

3.3. Stability analysis

To study the qualitative behavior of the system, we implement the stability analysis for the model (1).

Theorem 1. *The system* (1) *is locally asymptotically stable* (*LAS*) *at pest and parasite free equilibrium* E_0 *if* $\sqrt{4\alpha\gamma + (\gamma - \mu)^2} < \gamma + \mu$.

Proof. At E_0, the Jacobian matrix of the system is given by

$$J(E_0) = \begin{bmatrix} -\gamma & \alpha & 0 & 0 \\ \gamma & -\mu_1 & 0 & 0 \\ 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & -\mu_3 \end{bmatrix}.$$

The characteristic equation of the matrix is

$$(\gamma + \lambda)(\mu_1 + \lambda)(\mu_2 + \lambda)(\mu_3 + \lambda) - \alpha\gamma(\mu_2 + \lambda)(\mu_3 + \lambda) = 0.$$

The eigenvalues of the system are

$$\lambda = -\mu_3,\ -\mu_4,\ \frac{-\gamma - \mu}{2} \pm \frac{\sqrt{4\alpha\gamma + (\gamma - \mu)^2}}{2}.$$

The system has negative real roots if $\sqrt{4\alpha\gamma + (\gamma - \mu)^2} < \gamma + \mu$. Hence, E_1 is locally asymptotically stable if $\sqrt{4\alpha\gamma + (\gamma - \mu)^2} < \gamma + \mu$. □

Theorem 2. *The system* (1) *at the parasite free equilibrium* $\widetilde{E} = (\widetilde{x}_1, \widetilde{x}_2, \widetilde{P}, 0, \widetilde{A})$ *is LAS if* $R_4 > 0$, $R_1R_2 - R_3 > 0$ *and* $(R_1R_2 - R_3)R_3 - R_1^2R_4 > 0$.

Proof. The Jacobian matrix of the system at $\widetilde{E}$ is given by

$$J(\widetilde{E}) = \begin{bmatrix} \alpha\widetilde{x}_2\left(\frac{-1}{k}\right) - \gamma - \frac{(c^2+\widetilde{x}_1^2)(2a\widetilde{x}_1\widetilde{P})+2a\widetilde{x}_1^3\widetilde{P}}{(c^2+\widetilde{x}_1)^2} & \alpha\left(1 - \frac{\widetilde{x}_1+\widetilde{x}_2}{k}\right) - \frac{\alpha\widetilde{x}_2}{k} & -\frac{a\widetilde{x}_1^2}{c^2+\widetilde{x}_1^2} & -\omega\widetilde{x}_1 \\ \gamma & -\mu_1 & 0 & 0 \\ -\frac{(c^2+\widetilde{x}_1^2)(2a\widetilde{x}_1\widetilde{P})+2a\widetilde{x}_1^3\widetilde{P}}{(c^2+\widetilde{x}_1)^2} & 0 & \frac{ab\widetilde{x}_1^2}{c^2+\widetilde{x}_1^2} - \mu_2 & 0 \\ 0 & 0 & 0 & \omega\omega_1\widetilde{x}_1 - \mu_3 \end{bmatrix}.$$

The matrix $J(\widetilde{E})$ gives the characteristic equation $\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0$. The roots of the characteristic equation will have negative real parts when $R_4 > 0$, $R_1R_2 - R_3 > 0$ and $(R_1R_2 - R_3)R_3 - R_1^2R_4 > 0$. Hence, the system (1) around $\widetilde{E}$ is LAS if the Routh – Hurwitz (R-H) criterion is satisfied. □

Theorem 3. *The system* (1) *at the endemic equilibrium* $E_* = \left(x_1^*, x_2^*, P^*, S^*\right)$ *is LAS if* $Q_4 > 0$, $Q_1Q_2 - Q_3 > 0$ *and* $(Q_1Q_2 - Q_3)Q_3 - Q_1^2Q_4 > 0$.

Proof. The Jacobian matrix of the system at E_* is given by

$$J(E_*) = \\ = \begin{bmatrix} \alpha x_2^*\left(\frac{-1}{k}\right) - \omega S^* - \gamma - \frac{(c^2+x_1^{*2})(2ax_1^*P^*)+2ax_1^{*3}P^*}{(c^2+x_1^*)^2} & \alpha\left(1 - \frac{x_1^*+x_2^*}{k}\right) - \frac{\alpha x_2^*}{k} & -\frac{ax_1^{*2}}{c^2+x_1^{*2}} & -\omega x_1^* \\ \gamma & -\mu_1 & 0 & 0 \\ -\frac{(c^2+x_1^{*2})(2ax_1^*P^*)+2ax_1^{*3}P^*}{(c^2+x_1^*)^2} & \frac{abx_1^{*2}}{c^2+x_1^{*2}} - \mu_2 & 0 & \\ \omega\omega_1 S^* & 0 & 0 & \omega\omega_1 x_1^* - \mu_3 - 2\mu_4 S^* \end{bmatrix}.$$

The matrix $J(E_*)$ gives the characteristic equation $\lambda^4 + Q_1\lambda^3 + Q_2\lambda^2 + Q_3\lambda + Q_4 = 0$. The roots of the characteristic equation will have negative real parts when $Q_4 > 0$, $Q_1Q_2 - Q_3 > 0$ and $(Q_1Q_2 - Q_3)Q_3 - Q_1^2Q_4 > 0$. Hence, the system (1) around E_* is LAS if the R-H criterion is satisfied. □

Theorem 4. *The endemic equilibrium* E^* *is globally asymptotically stable* (*GAS*) *if the following inequalities hold*:

$$m_2 < \frac{\mu_4 S_m}{\omega\omega_1^2}, \quad \alpha < \frac{l\mu_1}{\zeta k},$$

$$m_1 < \frac{\alpha l}{\zeta k}\left(\mu_2 - \frac{l^2x_1^{*2}a + c^2abl^2}{\zeta^2\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}\right)\left(\frac{\zeta\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}{c^2ablP^* + c^2abP^*\zeta x_1^*}\right).$$

Proof. A Lyapunov function $V_0^*(x_1;\, x_2;\, P;\, S)$ in Ω is constructed as follows:

$$V_0^*(x_1;\, x_2;\, P;\, S) = \frac{1}{2}\left(x_1 - x_1^* + x_2 - x_2^*\right)^2 + \frac{m_1}{2}(P - P^*)^2 + m_2\left(S - S^*\ln\frac{S}{S^*}\right), \tag{5}$$

$$\frac{dV_0^*}{dt} = \left(x_1 - x_1^* + x_2 - x_2^*\right)\left(\frac{dx_1}{dt} + \frac{dx_2}{dt}\right) + m_1(P - P^*)\frac{dP}{dt} + m_2(S - S^*)\frac{dS}{dt}\frac{1}{S},$$

where m_1, m_2 are positive constants. $\frac{dV_0^*}{dt}$ is calculated along the solution of the system. After simplification, we get

$$\begin{aligned}\frac{dV_0^*}{dt} = &-\left(x_1 - x_1^*\right)^2\left(\frac{\omega S^* + \alpha x_2}{k} + \frac{c^2aP^*x_1}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} + \frac{c^2aP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) - \\ &-\left(x_1 - x_1^*\right)\left(x_2 - x_2^*\right)\left(-\alpha + \frac{\alpha\left(x_1 + 2x_2 + x_2^*\right)}{k} + \frac{c^2aP^*x_1}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} + \right.\\ &\left. + \mu_1 - \omega S^* + \frac{c^2aP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) - \left(x_2 - x_2^*\right)\left(\alpha + \mu_1 + \frac{\alpha\left(x_1 + x_2 + x_2^*\right)}{k}\right) - \\ &-\left(\omega x_1 - m_2\omega\omega_1\right)\left(x_1 - x_1^*\right)\left(S - S^*\right) - \left(x_2 - x_2^*\right)\left(P - P^*\right)\frac{ax_1^2x_1^{*2} + c^2ax_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \\ &-\left(x_1 - x_1^*\right)\left(P - P^*\right)\left(\frac{ax_1^2x_1^{*2}}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} + \frac{c^2ax_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \right.\\ &\left. - \frac{m_1c^2abx_1P^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \frac{m_1c^2abP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) - m_2\mu_4\left(S - S^*\right) - \\ &-\left(P - P^*\right)^2\left(\mu_2m_1 - \frac{m_1x_1^2x_1^{*2}a}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \frac{m_1c^2abx_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) - \omega x_1\left(S - S^*\right)\left(x_2 - x_2^*\right).\end{aligned}$$

Thus, inside the region of attraction, $\frac{dV_0^*}{dt}$ is negative-definite under the condition that:

$$m_2 < \frac{\mu_4 S_m}{\omega\omega_1^2}, \quad \alpha < \frac{l\mu_1}{\zeta k},$$

$$m_1 < \frac{\alpha l}{\zeta k}\left(\mu_2 - \frac{l^2x_1^{*2}a + c^2abl^2}{\zeta^2\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}\right)\left(\frac{\zeta\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}{c^2ablP^* + c^2abP^*\zeta x_1^*}\right).$$

It is seen that $\frac{dV_0^*}{dt} < 0$ and $\frac{dV_0^*}{dt} = 0$ iff, $x_1 = x_1^*$, $x_2 = x_2^*$, $P = P^*$ and $S = S^*$ in Ω. Using the Lyapunov – LaSalle theorem, we conclude that E_* is GAS. □

4. Mathematical analysis of the model (3)

4.1. Positivity and boundedness

From system (3) it can be seen that

$$\left.\frac{dx_1}{dt}\right|_{x_1=0} = \alpha x_2\left(1 - \frac{x_2}{k}\right) \geqslant 0, \quad \left.\frac{dx_2}{dt}\right|_{x_2=0} = \gamma x_1 \geqslant 0,$$

$$\left.\frac{dS}{dt}\right|_{P=0} = \beta \geqslant 0, \quad \left.\frac{dP}{dt}\right|_{S=0} = 0, \quad \left.\frac{dA}{dt}\right|_{A=0} = r_0 + hx_2 \geqslant 0.$$

The total pest population $N = x_1 + x_2$ satisfies

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) - \frac{ax_1^2 P}{c^2 + x_1^2} - \omega x_1 S - \mu_1 x_2,$$

$$\frac{dN}{dt} + \zeta x_1 + \zeta x_2 \leqslant \alpha x_2\left(1 - \frac{x_1 + x_2}{k}\right) + \zeta x_1 + \zeta x_2,$$

$$\frac{dN}{dt} + \zeta N \leqslant -\frac{\alpha x_2^2}{k} + (\alpha + \zeta)x_2.$$

It is seen that $-\frac{\alpha x_2^2}{k} + (\alpha + \zeta)x_2$ is quadratic in x_2 and its maximum value is $\frac{(\alpha+\zeta)^2 k}{4\alpha}$

$$\frac{dN}{dt} + \zeta N \leqslant l,$$

where $l = \frac{(\alpha+\zeta)^2 k}{4\alpha}$.

$$0 \leqslant N(t) \leqslant e^{-\zeta t}\left(N(0) - \frac{l}{\zeta}\right) + \frac{l}{\zeta}.$$

As $t \to \infty$, $N(t) \to \frac{l}{\zeta}$ since $\sup\limits_{t\to\infty} N(t) = \frac{l}{\zeta}$.

Since x_1 is bounded,

$$\frac{dP}{dt} + \mu_2 P \leqslant mP + \beta.$$

Then $\sup\limits_{t\to\infty} P(t) = \frac{\beta}{\mu_2 - m}$.

$$\frac{dS}{dt} + \mu_3 S \leqslant RS - \mu_4 S.$$

Thus, $\sup\limits_{t\to\infty} S(t) = \frac{\mu_4^2}{4(R-\mu_3)}$. Since x_2 is bounded, $\sup\limits_{t\to\infty} A(t) = \frac{r_0 + hL}{\eta}$.

The positive invariant set of the system (3) is given by

$$\Omega_1 = \left\{(x_1, x_2, P, S, A) \in \mathbb{R}_+^5 \mid 0 \leqslant x_1, x_2 \leqslant \frac{l}{\zeta}, P \leqslant \frac{\beta}{\mu_2 - m}, S \leqslant \frac{\mu_4^2}{4(R - \mu_3)}, A \leqslant \frac{r_0 + hL}{\eta}\right\}.$$

4.2. Equilibrium points

The system (3) possesses three nonnegative equilibrium points. They are:

- pest and parasite free equilibrium: $E_1 = \left(0, 0, \frac{\beta}{\mu_2}, 0, \frac{r_0}{\eta}\right)$;
- parasite free equilibrium: $\overline{E} = (\overline{x}_1, \overline{x}_2, \overline{P}, 0, \overline{A})$ where

$$\overline{P} = \frac{\beta\left(c^2 + \left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right)^2\right)}{\mu_2\left(c^2 + \left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right)^2\right) - ab\left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right)^2}, \quad \overline{x}_2 = \frac{\eta\overline{A}}{r_0}, \quad \overline{x}_1 = \frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2,$$

$\overline{A}$ is the positive root of the equation:

$$\frac{\alpha\eta\overline{A}}{r_0}\left(1 - \left(\frac{\mu\eta\overline{A}}{\gamma k r_0} + \frac{\delta\eta\overline{A}^2}{\gamma k r_0} + \frac{\eta\overline{A}}{k r_0}\right)\right) - \frac{a\overline{P}\left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right)}{c^2 + \left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right)^2} - \gamma\left(\frac{\mu\eta}{\gamma r_0}\overline{A} + \frac{\delta\eta}{\gamma r_0}\overline{A}^2\right) = 0;$$

- endemic equilibrium: $E_* = \left(x_1^*, x_2^*, P^*, S^*, A^*\right)$ where

$$P^* = \frac{\beta\left(c^2 + \left(\frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}\right)^2\right)}{\mu_2\left(c^2 + \left(\frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}\right)^2\right) - ab\left(\frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}\right)^2},$$

$$x_2^* = \frac{\eta A^*}{r_0}, \quad x_1^* = \frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}, \quad S^* = \frac{\omega\omega_1}{\mu_4}\left(\frac{\mu\eta A^*}{\gamma r_0} + \frac{\delta\eta A^{*2}}{\gamma r_0}\right) - \frac{\mu_3}{\mu_4},$$

A^* is the positive root of the equation:

$$\frac{\alpha\eta A^*}{r_0}\left(1 - \left(\frac{\mu\eta A^*}{\gamma k r_0} + \frac{\delta\eta A^{*2}}{\gamma k r_0} + \frac{\eta A^*}{k r_0}\right)\right) - \frac{aP^*\left(\frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}\right)}{c^2 + \left(\frac{\mu\eta}{\gamma r_0}A^* + \frac{\delta\eta}{\gamma r_0}A^{*2}\right)^2} - \omega x_1^* S^* - \gamma x_1^* = 0.$$

4.3. Stability analysis

To study the qualitative behavior of dynamical system, we perform a stability analysis for the model (3).

Theorem 5. *The system* (3) *is locally asymptotically stable at* E_1 *if*

$$\sqrt{4\alpha\eta^2\gamma + (\eta\gamma + \delta r_0)^2 + \eta^2\mu^2 + 2\eta\mu(\delta r_0 - \eta\gamma)} < \eta\gamma + r_0\delta + \mu\eta.$$

Proof. At E_1, the Jacobian matrix of the system is given by

$$J(E_1) = \begin{bmatrix} -\gamma & \alpha & 0 & 0 & 0 \\ \gamma & -\mu_1 - \frac{\delta r_0}{\eta} & 0 & 0 & 0 \\ 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & -\mu_3 & 0 \\ 0 & h & 0 & 0 & -\eta \end{bmatrix}.$$

The characteristic equation of the matrix is

$$(\gamma + \lambda)\left(\mu_1 + \frac{\delta r_0}{\eta} + \lambda\right)(\mu_2 + \lambda)(\mu_3 + \lambda)(\eta + \lambda) - \alpha\gamma(\mu_2 + \lambda)(\mu_3 + \lambda)(\eta + \lambda) = 0.$$

The eigenvalues of the system are

$$\lambda = -\mu_2,\ -\mu_3,\ -\eta,\ -\frac{\eta\gamma + r_0\delta + \mu\eta}{2\eta} \pm \frac{\sqrt{4\alpha\eta^2\gamma + (\eta\gamma + \delta r_0)^2 + \eta^2\mu^2 + 2\eta\mu(\delta r_0 - \eta\gamma)}}{2\eta}.$$

The system has negative real roots if

$$\sqrt{4\alpha\eta^2\gamma + (\eta\gamma + \delta r_0)^2 + \eta^2\mu^2 + 2\eta\mu(\delta r_0 - \eta\gamma)} < \eta\gamma + r_0\delta + \mu\eta.$$

Hence, E_1 is locally asymptotically stable. □

Theorem 6. *The system* (3) *at the parasite free equilibrium* $\overline{E} = (\widetilde{x}_1, \overline{x}_2, \overline{P}, 0, \overline{A})$ *is LAS if* $r_i > 0$ $(i = 1, \ldots, 5)$, $r_1r_2r_3 > r_3^2 + r_1^2r_4$ *and* $(r_1r_4 - r_5)\left(r_1r_2r_3 - r_3^2 - r_1^2r_4\right) > r_5(r_1r_2 - r_3)^2 + r_1r_5^2$.

Proof. The Jacobian matrix of the system at $\overline{E}_1$ is given by

$$J(\overline{E}_1) = \begin{bmatrix} \alpha\overline{x}_2\left(\frac{-1}{k}\right) - \gamma - \frac{\left(c^2+\overline{x}_1^2\right)(2a\overline{x}_1\overline{P})+2a\overline{x}_1^3\overline{P}}{\left(c^2+\overline{x}_1\right)^2} & \alpha\left(1 - \frac{\overline{x}_1+\overline{x}_2}{k}\right) - \frac{\alpha\overline{x}_2}{k} & -\frac{a\overline{x}_1^2}{c^2+\overline{x}_1^2} & -\omega\overline{x}_1 & 0 \\ \gamma & -\mu_1 - \delta\overline{A} & 0 & 0 & -\delta\overline{x}_2 \\ -\frac{\left(c^2+\overline{x}_1^2\right)(2a\overline{x}_1\overline{P})+2a\overline{x}_1^3\overline{P}}{\left(c^2+\overline{x}_1\right)^2} & 0 & \frac{ab\overline{x}_1^2}{c^2+\overline{x}_1^2} - \mu_2 & 0 & 0 \\ 0 & 0 & 0 & \omega\omega_1\overline{x}_1 - \mu_3 & 0 \\ 0 & h & 0 & 0 & -\eta \end{bmatrix}.$$

The matrix $J(\overline{E})$ gives the characteristic equation $\lambda^5 + r_1\lambda^4 + r_2\lambda^3 + r_3\lambda^2 + r_4\lambda + r_5 = 0$. The roots of the characteristic equation will have negative real parts when $r_i > 0$ $(i = 1, \ldots, 5)$, $r_1r_2r_3 > r_3^2 + r_1^2r_4$ and $(r_1r_4 - r_5)\left(r_1r_2r_3 - r_3^2 - r_1^2r_4\right) > r_5(r_1r_2 - r_3)^2 + r_1r_5^2$. Hence, the system (1) around $\widetilde{E}$ is LAS. □

Theorem 7. *The system* (3) *at the endemic equilibrium* $E_* = \left(x_1^*, x_2^*, P^*, S^*, A^*\right)$ *is LAS if* $q_i > 0$ $(i = 1, \ldots, 5)$, $q_1q_2q_3 > q_3^2 + q_1^2q_4$ *and* $(q_1q_4 - q_5)\left(q_1q_2q_3 - q_3^2 - q_1^2q_4\right) > q_5(q_1q_2 - q_3)^2 + q_1q_5^2$.

Proof. The Jacobian matrix of the system at E_* is given by

$$J(E_*) =$$

$$= \begin{bmatrix} \alpha x_2^*\left(\frac{-1}{k}\right) - \omega S^* - \gamma - \frac{\left(c^2+x_1^{*2}\right)\left(2ax_1^*P^*\right)+2ax_1^{*3}P^*}{\left(c^2+x_1^*\right)^2} & \alpha\left(1 - \frac{x_1^*+x_2^*}{k}\right) - \frac{\alpha x_2^*}{k} & -\frac{ax_1^{*2}}{c^2+x_1^{*2}} & -\omega x_1^* \\ \gamma & -\mu_1 & 0 & 0 \\ -\frac{\left(c^2+x_1^{*2}\right)\left(2ax_1^*P^*\right)+2ax_1^{*3}P^*}{\left(c^2+x_1^*\right)^2} & \frac{abx_1^{*2}}{c^2+x_1^{*2}} - \mu_2 & 0 & \\ \omega\omega_1S^* & 0 & 0 & \omega\omega_1x_1^* - \mu_3 - 2\mu_4S^* \end{bmatrix}.$$

The matrix $J(E_*)$ gives the characteristic equation $\lambda^5 + q_1\lambda^4 + q_2\lambda^3 + q_3\lambda^2 + q_4\lambda + q_5 = 0$. The roots of the characteristic equation will have negative real parts when $q_i > 0$ $(i = 1, \ldots, 5)$, $q_1q_2q_3 > q_3^2 + q_1^2q_4$ and $(q_1q_4 - q_5)\left(q_1q_2q_3 - q_3^2 - q_1^2q_4\right) > q_5(q_1q_2 - q_3)^2 + q_1q_5^2$. Hence, the system (1) around E_* is LAS. □

Theorem 8. *The endemic equilibrium* E_* *is globally asymptotically stable* (*GAS*) *if the following inequalities hold*:

$$n_2 < \frac{\mu_4 S_m}{\omega\omega_1^2}, \quad \alpha < \frac{l\mu_1}{\zeta k}, \quad h^2 < \delta A_m \eta,$$

$$n_1 < \frac{\alpha l}{\zeta k}\left(\mu_2 - \frac{l^2x_1^{*2}a + c^2abl^2}{\zeta^2\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}\right)\left(\frac{\zeta\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}{c^2ablP^* + c^2abP^*\zeta x_1^*}\right).$$

Proof. A Lyapunov function $V_1^*(x_1; x_2; P; S; A)$ in Ω_1 is constructed as follows:

$$V_1^*(x_1; x_2; P; S; A) = \frac{1}{2}\left(x_1 - x_1^* + x_2 - x_2^*\right)^2 + \frac{n_1}{2}(P - P^*)^2 + n_2\left(S - S^* - S^*\ln\frac{S}{S^*}\right) + \frac{1}{2}(A - A^*)^2, \tag{6}$$

$$\frac{dV_1^*}{dt} = \left(x_1 - x_1^* + x_2 - x_2^*\right)\left(\frac{dx_1}{dt} + \frac{dx_2}{dt}\right) + n_1(P - P^*)\frac{dP}{dt} + n_2(S - S^*)\frac{dS}{dt}\frac{1}{S} + (A - A^*)\frac{dA}{dt},$$

where n_1, n_2 are positive constants. $\frac{dV_1^*}{dt}$ is calculated along the solution of the system. After simplification, we get

$$\begin{aligned}
\frac{dV_1^*}{dt} = &-\left(x_1 - x_1^*\right)^2\left(\frac{\omega S^* + \alpha x_2}{k} + \frac{c^2aP^*x_1}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} + \frac{c^2aP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) -\\
&-\left(x_1 - x_1^*\right)\left(x_2 - x_2^*\right)\left(-\alpha + \frac{\alpha\left(x_1 + 2x_2 + x_2^*\right)}{k} - \omega S^* + \frac{c^2aP^*x_1}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} +\right.\\
&\left.+\mu_1 + \frac{c^2aP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) - \left(x_2 - x_2^*\right)\left(\alpha + \mu_1 + \frac{\alpha\left(x_1 + x_2 + x_2^*\right)}{k}\right) -\\
&-\left(\omega x_1 - m_2\omega\omega_1\right)\left(x_1 - x_1^*\right)\left(S - S^*\right) - \left(x_2 - x_2^*\right)\left(P - P^*\right)\frac{ax_1^2x_1^{*2} + c^2ax_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} -\\
&-\left(x_1 - x_1^*\right)\left(P - P^*\right)\left(\frac{ax_1^2x_1^{*2}}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} + \frac{c^2ax_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} -\right.\\
&\left.-\frac{m_1c^2abx_1P^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \frac{m_1c^2abP^*x_1^*}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) + h\left(A - A^*\right)\left(x_2 - x_2^*\right) -\\
&-\delta x_2\left(A - A^*\right)\left(x_1 - x_1^*\right) - \delta A^*\left(x_2 - x_2^*\right)\left(x_1 - x_1^*\right) - \delta A\left(x_2 - x_2^*\right) - \eta\left(A - A^*\right) -\\
&-\left(P - P^*\right)^2\left(\mu_2m_1 - \frac{m_1x_1^2x_1^{*2}a}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)} - \frac{m_1c^2abx_1^2}{\left(c^2 + x_1^2\right)\left(c^2 + x_1^{*2}\right)}\right) -\\
&-\omega x_1\left(S - S^*\right)\left(x_2 - x_2^*\right) - m_2\mu_4\left(S - S^*\right).
\end{aligned}$$

Thus, inside the region of attraction, $\frac{dV_1^*}{dt}$ is negative-definite under the condition that:

$$n_2 < \frac{\mu_4 S_m}{\omega\omega_1^2}, \quad \alpha < \frac{l\mu_1}{\zeta k}, \quad h^2 < \delta A_m\eta,$$

$$n_1 < \frac{\alpha l}{\zeta k}\left(\mu_2 - \frac{l^2x_1^{*2}a + c^2abl^2}{\zeta^2\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}\right)\left(\frac{\zeta\left(c^2 + \left(\frac{l}{\zeta}\right)^2\right)\left(c^2 + x_1^{*2}\right)}{c^2ablP^* + c^2abP^*\zeta x_1^*}\right).$$

It is seen that $\frac{dV_1^*}{dt} < 0$ and $\frac{dV_1^*}{dt} = 0$ iff, $x_1 = x_1^*$, $x_2 = x_2^*$, $P = P^*$, $S = S^*$ and $A = A^*$ in Ω_1. Using the Lyapunov – LaSalle theorem, we conclude that E_* is GAS. □

5. Numerical simulation

The numerical analysis is carried out using the parameter values as given in Table 1. The initial conditions are assumed to be $x_1(t) = 500$, $x_2(t) = 50$, $P(t) = 10$, $S(t) = 10$, and $A(t) = 5$. The values corresponding to the parameters are calculated based on experimental data and values given in the literature [Elango, Nelson, Aravind, 2020; Alagar et al., 2020; Elango, Nelson, 2020; Suriya et al., 2021; Rao, Ramani, Bhagavan, 2020; Chalapathi Rao et al., 2022]. The carrying capacity of pest per leaf is approximately considered as 5000 leaf^{-1}. The fecundity rate of pest is calculated from the value given in [Elango, Nelson, Aravind, 2020]. The predation rate, its birth and the death rate of predator are calculated from [Rao, Ramani, Bhagavan, 2020; Chalapathi Rao et al., 2022]. The parasitization rate, its birth and death rate are calculated based on the data given in [Elango, Nelson, 2020; Suriya et

Table 1. Parameter values used for analysis which are calculated based on [Elango, Nelson, Aravind, 2020; Alagar et al., 2020; Elango, Nelson, 2020; Suriya et al., 2021; Rao, Ramani, Bhagavan, 2020; Chalapathi Rao et al., 2022]

Symbol	Meaning	Unit	Value taken for analysis
K	Carrying capacity of leaf	Leaf^{-1}	5000
α	Egg laying rate of mature pest	pest^{-1}day^{-1}	0.03–0.05
a	Predation rate	day^{-1}	0.019
c	Saturation constant		0.1
ω	Parasitization rate	parasite^{-1}day^{-1}	0.0004–0.001
γ	Conversion to mature population	day^{-1}	0.02–0.03
μ_1	Mortality rate of whitefly	pest^{-1}	0.005–0.02
b	Conversion rate of predator	day^{-1}	0.58–0.78
μ_2	Mortality rate of predator	day^{-1}	0.02–0.04
β	no. of predator added to the system	day^{-1}	1
ω_1	Conversion rate of parasite	day^{-1}	0.2–0.6
μ_3	Mortality rate of parasite	day^{-1}	0.01–0.02
δ	Mortality of whitefly due to awareness activity	day^{-1}	0.005
μ_4	Density-dependent death rate of parasite	day^{-1}	0.0005
r	Rate of awareness programs	day^{-1}	0.03
h	Rate of local awareness	day^{-1}	0.025
η	Fading rate of awareness	day^{-1}	0.015

al., 2021]. The awareness related parameter values are taken from [Basir, Banerjee, Ray, 2019; Basir, Blyuss, Ray, 2018; Basir et al., 2018].

Figure 1 shows the effect of predation rate and parasitization rate of immature pest $x_1(t)$ in system (1). The population decreases for both the rates. Figure 2 infers the mature population $x_2(t)$ in system (1) for varying values of γ and ω.

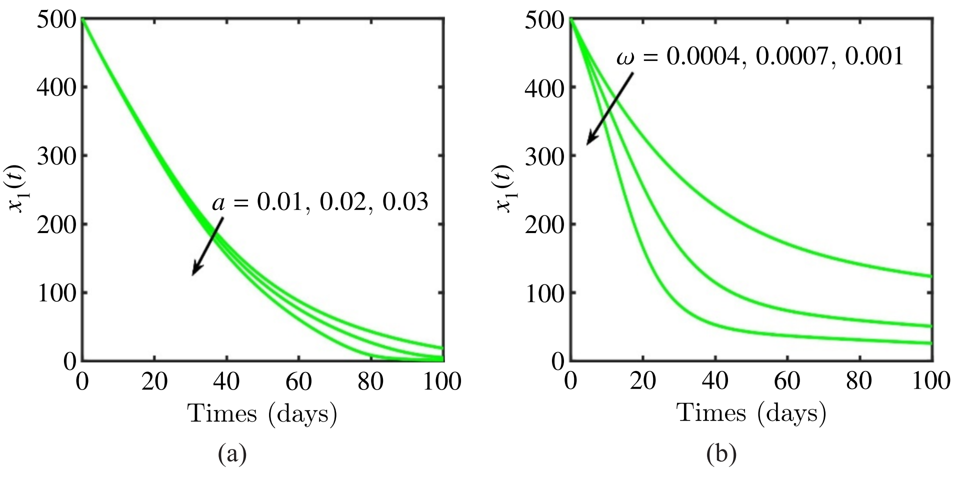

Figure 1. Plot of immature pest population $x_1(t)$ versus time (days) for system (1) (a) for different values of predation rate a (b) for different values of parasitization rate ω with other parameter values as given in Table 1

The increase in conversion rate increases the adult pest population and the increased effect of parasite in immature pest reduces the mature population. From Figure 3, we see that the predation rate increases the predator population and the mortality rate μ_2 reduces the predator in system (1). Figure 4

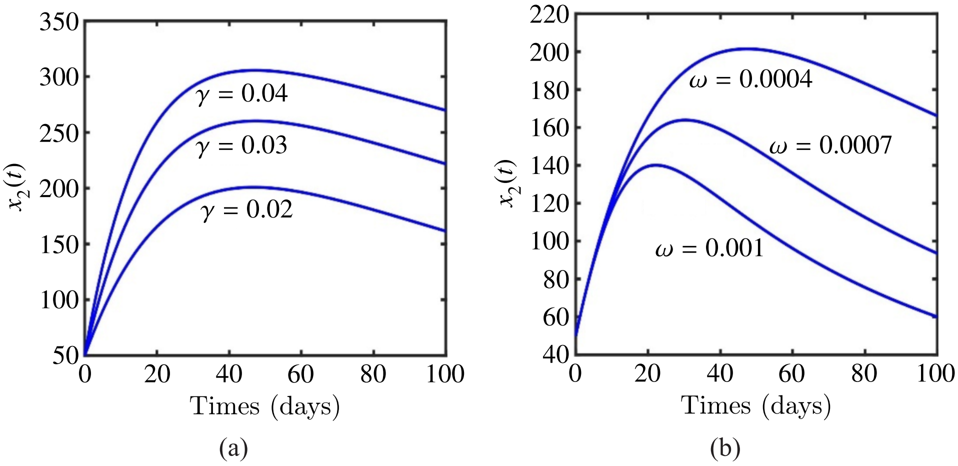

Figure 2. Plot of mature pest population $x_2(t)$ versus time for system (1) (a) variation due to conversion rate γ (b) variation due to ω with other parameter values as given in Table 1

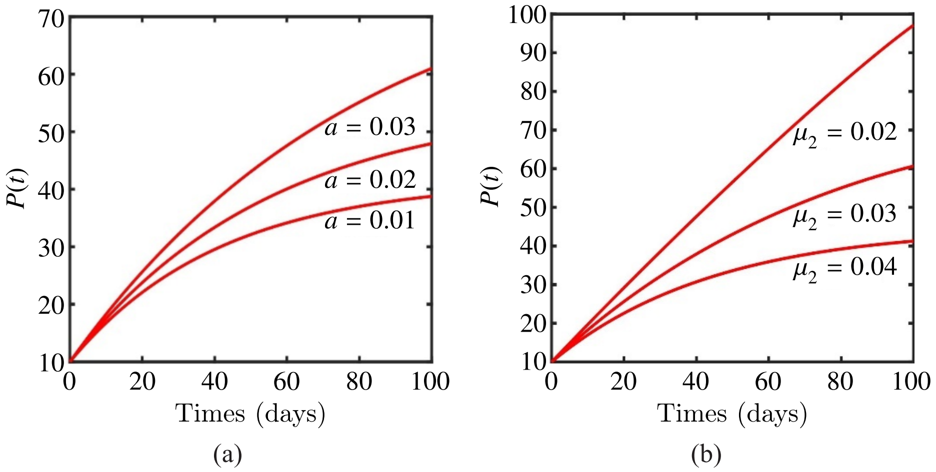

Figure 3. Profile of predator population $P(t)$ versus time (days) for system (1) (a) variation due to predation rate a (b) variation due to its mortality rate μ_2

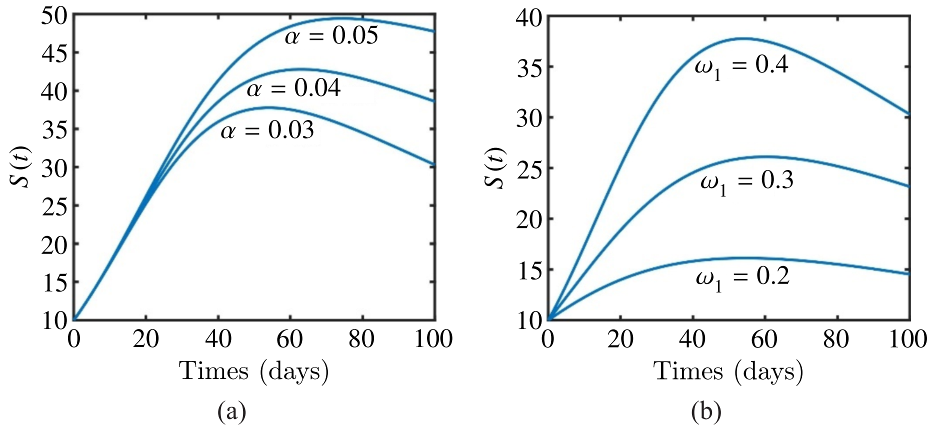

Figure 4. Profile of parasite $S(t)$ versus time (days) for system (1) (a) variation due to α (b) for different values of its growth rate $\omega_1(t)$

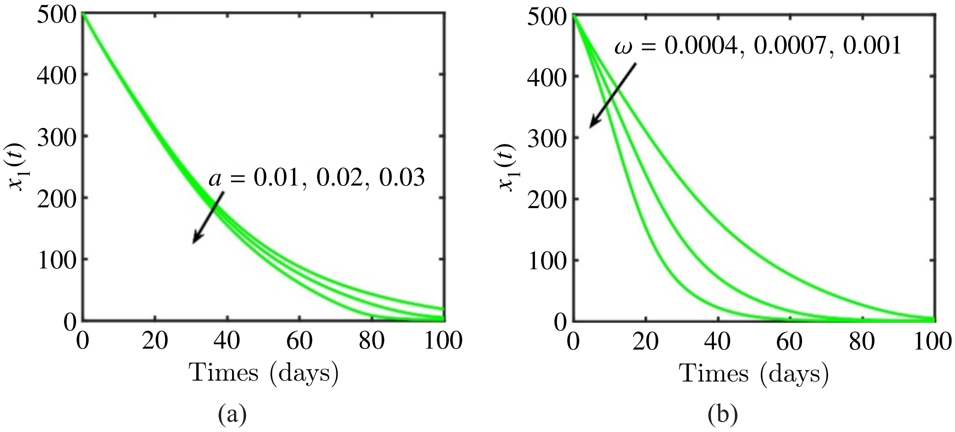

Figure 5. Plot of immature pest $x_1(t)$ versus time for system (3) (a) for different values of a (b) for different values of parasitization rate ω with other parameters as given in Table 1

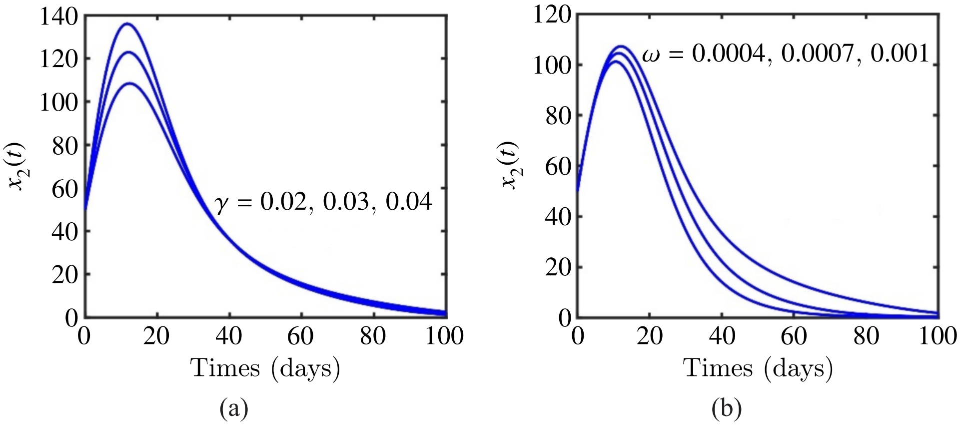

Figure 6. Plot of mature population $x_2(t)$ versus time for system (3) (a) variation due to γ (b) variation due to ω

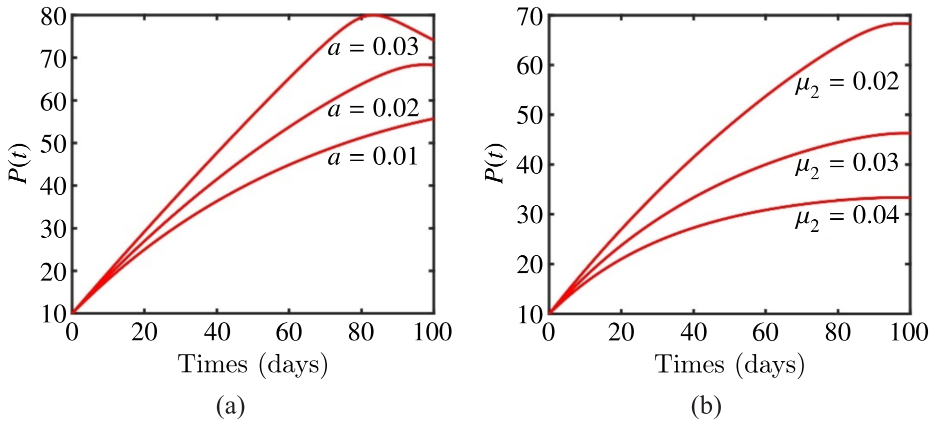

Figure 7. Plot of predator population $P(t)$ versus time for system (3) (a) for different values of predation rate a (b) for different values of its death rate μ_2 with other parameters as given in Table 1

interprets the plot of parasite $S(t)$ as it increases due to increased values of egg laying rate α and conversion rate ω_1. In Figure 5, we see the effect of predation rate and parasite in immature pest in the presence of awareness for system (3). Figure 6 depicts the mature population due to γ and ω. It

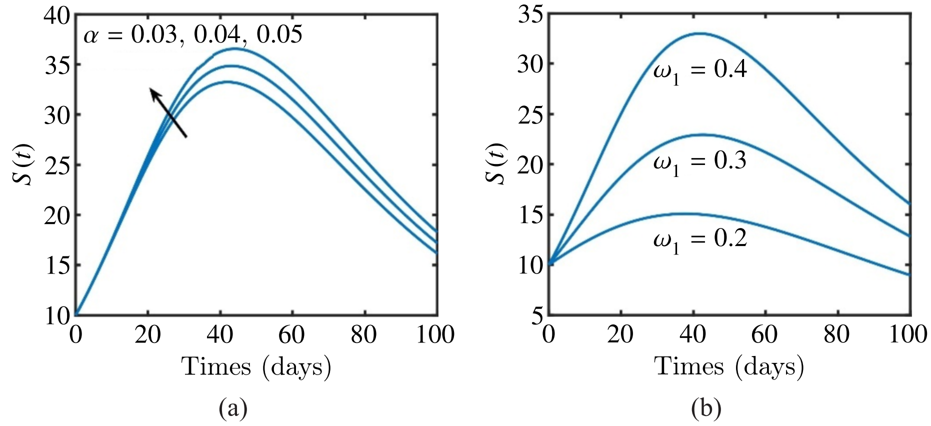

Figure 8. Profile of parasite $S(t)$ versus time (days) for system (3) (a) variation due to α (b) for different values of its conversion rate $\omega_1(t)$

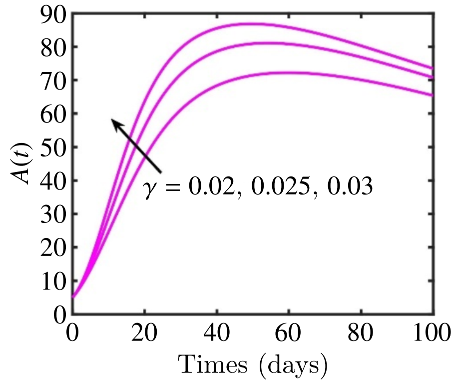

Figure 9. Profile of awareness programs $A(t)$ versus time for different values of γ

shows a decreased population compared to the mature population in system (1). Figure 7 infers the predator population $P(t)$ in system (3) for variation in its growth rate and death rate. From Figure 8, we see that the parasite population in system (3) increases for α and ω_1. Figure 9 shows that the increase in mature population is proportional to the awareness programs $A(t)$. From these figures, it is implied that the predator and the parasite effectively reduce the immature population in system (1). On incorporating awareness programs, there is mortality in mature pest due to awareness activities. Hence, in system (3), we see that mature population is reduced by which the egg laying rate is decreased. This results in decreased pest population compared to the system without awareness. Thus, the pest population can be reduced using biocontrol agents like predator and parasite. Also, educating the farmers through awareness campaigns and media activities helps to supress the pest population and spread of the disease.

6. Conclusion

A more realistic mathematical model has been developed and analyzed to assess the capacity of biological agents on RSW pest control in coconut trees. Further extension has been done by incorporating the impact of awareness programs which resulted in much better control over pest. The feasible steady states and the stability conditions are analyzed. The effect of individual parameter on the species population is studied via numerical simulation. From the above results, it is evident that the presence of predator and parasite controls the immature pest population to a greater extent.

The awareness parameter in the model deals with the farmer's awareness on control measures. If the awareness through media or agri campaigns are efficient enough to reach each and every farmer, then it makes more positive effect on controlling the mature fly population. At the end, by the combined effects of biologcal agents to control immature flies and control techniques to curb mature flies resulted in better pest control such that the yield of the tree doesn't get affected.

Conflict of interest

The authors declare that there is no conflict of interest.

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